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Dividend Strips

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Disaster Recovery and the Term Structure of Dividend Strips

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Abstract

Recent empirical findings document downward-sloping term-structures of equity volatility and risk premia. An equilibrium model with rare disasters followed by recovery helps reconcile theory with empirical observations. While previous models focus on frequency and size of disasters, we show that recovery from disasters, a feature of the data, is at least as important. Indeed, recoveries outweigh the upward-sloping effect of time-varying disaster intensity, generating downward-sloping term-structures of dividend risk, equity risk, and equity risk premia. The model quantitatively reconciles a high equity premium and a low risk-free rate with downward-sloping term-structures, which are at odds in standard frameworks.

Keywords: dividend strips, recoveries, term-structure of risk and return, time-varying rare disasters

JEL Classification. D51, E21, G12

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1 Introduction

Recent empirical studies show that the term-structures of dividend risk, equity return volatility and equity risk premia are downward-sloping (van Binsbergen, Brandt, and Koijen, 2012). These findings are particularly important because they question the validity of the most successful asset-pricing models. In particular, the time-varying disaster risk models developed by Gabaix (2012) and Wachter (2013) provide a convincing foundation for the observed levels and dynamics of equity volatility and risk premia, but implies term-structures of equity volatility and risk premia that are inconsistent with the data.

In this paper we account for the empirically supported fact that dividends recover after disasters. While either natural or man-made disasters affect both physical and, to a lower extent, human capital, it is easy to understand disaster recovery by means of knowledge conservation. Available technology and know-how allow to speed-up post-disaster economic growth. Indeed, capital accumulation is easier the second time around because it replicates a known investment pattern. Moreover, disasters induce government spending to stimulate the economic environment and to foster competition.¹

We show that disaster recovery helps understand the shape of the term-structures of equity. Indeed, we provide theoretical evidence that recoveries kill the upward-sloping effect of time-varying risk of rare disaster and therefore imply empirically consistent term-structures of equity. The reason is that, in the presence of recoveries, the volatility of dividends is larger in the short-term than in the long-term. In equilibrium, the properties of dividend volatility transmit to stock returns and imply downward-sloping term-structures of equity volatility and risk premia.

We consider a pure-exchange economy (Lucas, 1978) with a representative investor who has Epstein and Zin (1989) preferences. As usual, the investor chooses a consumption plan and a portfolio invested in one stock and one riskless asset to maximize his expected lifetime utility. In equilibrium, the investor consumes the dividend paid by the stock and invests his entire wealth in it. The key feature of our model is that rare disasters might hit dividends. We assume that the intensity of disaster arrivals is time-varying (Wachter, 2013) and that recoveries take place right after the occurrence of a disaster (Gourio, 2008).

We show that the presence of recoveries implies high dividend volatility in the short-term and low dividend volatility in the long-term. The reason is that, in the short-term (e.g., 1 day), the dividend incurs the risk of a disaster, but the horizon is too short to benefit from a significant recovery. In the long-term (e.g., 20 years), however, disaster risk is still present, but

¹While there is a debate about the short-run and long-run impacts of disasters on economic growth, developed economies, such as the U.S., seem to be able to mitigate adverse effects and exploit growth opportunities. See Cavallo, Galiani, Noy, and Pantano (2013) and references therein.

dividends have a significant amount of time to recover. We characterize conditions such that, in equilibrium, stock returns inherit the dynamics of dividend growth rates. Therefore, the term-structure of equity volatility is, as the term-structure of dividend volatility, downward-sloping in our model. Under standard preferences (elasticity of intertemporal substitution larger than one), the timing of the equilibrium compensation required by the representative investor follows equity volatility and, in turn, the term-structure of equity risk premia is also downward-sloping.

To understand the dynamic patterns of the term-structures of equity over the business cycle, we define bad (resp., good) economic times as states of the world in which the disaster intensity is high (resp., low). Consistent with [van Binsbergen, Hueskes, Kojen, and Vrugt \(2013\)](#), we show that the slopes of the term-structures of equity are pro-cyclical in our model, being smaller in bad times than in good times. The reason is that, in bad times, the disaster intensity is large and is consequently expected to revert back down to its mean in the long-term. This implies a significantly larger amount of risk in the short term than in the long term, and therefore steep downward-sloping term-structures of equity. In good times, however, the disaster intensity is small and will eventually revert back up to its long-term mean. That is, disaster intensity risk is larger in the long-term than in the short-term. In equilibrium, this mitigates the downward-sloping effect of recoveries and implies flat term-structures of equity.

Accounting for recoveries helps explain the shape of the term-structure of equity, but it has the undesired consequence of significantly decreasing the level of the risk premium. To resolve this issue, we extend our model and assume that, conditional on a disaster, dividends drop more significantly and recover faster than total output. This assumption is motivated by our empirical exercise, which provides evidence that aggregate U.S. dividends are more strongly struck by disasters and recover faster than the U.S. aggregate consumption (or GDP). We also find that dividends scaled by consumption are stationary, and therefore we assume that they are co-integrated in our model. In addition, we follow [Bansal and Yaron \(2004\)](#) and assume that the expected growth rate of dividends and consumption is mean-reverting and reasonably persistent. We argue that, even though long-run risk and time-varying risk of rare disasters imply empirically inconsistent term-structures, extending the latter model with plausible recoveries helps explain simultaneously several important properties of dividends, consumption, and asset prices.²

First, the term-structure of consumption volatility is upward-sloping, whereas that of dividend volatility is downward-sloping ([Marfè, 2013](#)). The former occurs because the consumption's speed of recovery is not high enough to outweigh time-varying risk of disasters and

²Consistent with the international evidence documented by [Gourio \(2008\)](#), our calibration implies that dividends recover after large drops in about five years. This is the joint result of long-run growth and after-disaster excessive conditional growth.

long-run risk, both of which imply a large amount of risk in the long-term. The term-structure of dividend volatility, however, is downward-sloping because dividends recover sufficiently fast to be safer in the long-term than in the short-term. We provide empirical support for such a mechanism. Co-integration requires that consumption and dividends face the same permanent shock. However, the stationary dividend-share of consumption moves negatively with disasters and positively with recoveries. This implies that disasters are transitory shocks and that dividends load more on both disaster risk and recovery than consumption does. Consequently, the levered exposition of dividends to transitory risk helps explain the gap between the upward-sloping term-structure of consumption risk and the downward-sloping term-structure of dividend risk.

Second, the term-structures of equity are downward-sloping, except in good times where they are slightly upward-sloping. As explained earlier, the slopes of the term-structures are largest in good times because, in that state, the disaster intensity is expected to revert back up to its long-term mean. This creates more risk in the long-term than in the short-term and therefore larger slopes than in any other states. In our benchmark model the term-structures are flat in good times, yet they are slightly increasing in the extended model because of the presence of long-run risk.

Third, the aforementioned properties of the term-structures of equity risk premia and volatility hold even for an elasticity of intertemporal substitution smaller than one.³ The reason is that stock returns inherit the dynamic properties of dividends as long as the EIS is larger than some lower bound. This lower bound is equal to one for the consumption claim, whereas it turns out to be smaller than one for the dividend claim. This occurs because the empirical property that the dividend-share of consumption moves with disasters and recoveries implies a levered exposition of dividends on disaster risk.

Fourth, several asset-pricing moments are in line with the empirical findings. Indeed, the risk-free rate is about 1% and its volatility 3%. The equity risk premium is about 7%. Interestingly, the model generates a relatively large risk premium because, in the presence of recoveries, the risk premium increases when the elasticity of intertemporal substitution decreases (Gourio, 2008).⁴ Finally, stock-return volatility is about 10%, somewhat lower than in the data because, for the sake of exposition, we do not include idiosyncratic risk in the dividend dynamics.

Fifth, the ability of the model to solve the risk-free rate and equity premium puzzle and simultaneously capture the negative slopes of the term-structures of equity is robust to the

³A major critique of long-run risk and rare disasters models is their reliance on large elasticity of intertemporal substitution (Epstein, Farhi, and Strzalecki, 2014). As the estimations performed by Hall (1988) show, the elasticity of intertemporal substitution is likely to be in fact smaller than one.

⁴Note that if disasters are permanent, then the risk premium increases with the elasticity of intertemporal substitution.

setting of investors preferences. In particular, we show that the result can be obtained for any value of elasticity of intertemporal substitution in the range $(1/2, 2)$ and for low relative risk aversion.

Our model builds on the literature about time-varying risk of rare disasters. Such models provide a theoretical explanation of a number of price patterns (Gabaix (2012), Wachter (2013)) and has found empirical support (as in Berkman, Jacobsen, and Lee (2011) among others). We complement this literature by pointing out the importance of recovery (Gourio (2012)) and focusing on the otherwise puzzling term-structures of both fundamentals and equity. A few recent papers provide equilibrium explanations which help to explain the term-structure of equity. Ai, Croce, Diercks, and Li (2012), Belo, Collin-Dufresne, and Goldstein (2014) and Marfè (2014) focus respectively on investment risk, financial leverage and labor relations: these macroeconomic channels contribute to endogenize the downward-sloping term-structure of dividend risk and, in turn, of equity. This paper differs from the aforementioned studies because it focuses on an endowment economy where dividends feature time-varying disaster risk and recover after the occurrence of a disaster. The model calibration provides support to the idea that the disaster-recovery channel has a potentially sizeable quantitative impact on asset price dynamics. Other theoretical studies that help understand the term-structures of equity focus on non-standard specifications of beliefs formation and preferences. These are proposed by Croce, Lettau, and Ludvigson (2012) and Berrada, Detemple, and Rindisbacher (2013), respectively. In addition, Lettau and Wachter (2007, 2009) show in a partial equilibrium setting that a pricing kernel which enhances short-run risk can help to simultaneously explain the term-structure of equity and the cross-sectional value premium. Similar to Marfè (2014), our equilibrium framework allows to endogenize the pricing kernel and to provide an economic rationale for the intuition provided in Lettau and Wachter (2007, 2009). Similar to Longstaff and Piazzesi (2004), we assume in the extension of our model that the dividend-share of consumption is stationary. While they focus on the relation between the dynamics of corporate cash flows and the equity premium, our focus is on the impact of recoveries and recursive preferences on the term-structures of equity.

The remainder of the paper is organized as follows: Section 2 provides empirical support to our main assumptions; Section 3 presents and solves the model; Sections 4 and 5 describe the implications of recoveries on the term-structures of dividend risk, equity risk premia, and equity volatility; Section 6 extends the model to account for long-run risk and for the fact that, conditional on a disaster, dividends drop more and recover faster than consumption; and Section 7 concludes. Derivations are provided in Appendix A.

2 Empirical Support

Gourio (2008) provides evidence that rare disasters are followed by recovery. This means that, after large drops, GDP and consumption growth feature a conditional mean that is larger than the unconditional one. Such evidence suggests that, in contrast with what is usually considered in the literature, rare disasters should be modelled as transitory shocks instead of permanent shocks. Whether disasters have transitory or permanent nature is key to the extent of modelling the term-structure of risk of macroeconomic fundamentals, i.e., the volatility of growth rates computed over different time-horizons or the corresponding variance-ratios.

Belo, Collin-Dufresne, and Goldstein (2014) and Marfè (2013, 2014) document that aggregate dividends are characterized by a markedly downward-sloping term-structure of risk, whereas aggregate consumption features a slightly upward-sloping term-structure. The corresponding variance-ratios are reported in the upper panel of Figure 1. The co-integrating relationship between consumption and dividends allows us to interpret their different term-structures of risk as follows. Since consumption and dividends are co-integrated, they share the same permanent shock, whereas they are allowed to feature a different exposure to a transitory component. The former yields an upward-sloping effect due for instance to time-variation in long-run growth, whereas the latter yields a downward-sloping effect due to its mean-reverting dynamics. Because consumption loads to a lesser extent on the transitory shock than dividends do, the upward-sloping effect dominates and the term-structure of risk increases for consumption growth. Dividends, instead, load to a larger extent on the transitory shock, implying a dominance of the downward-sloping effect and therefore a decreasing term-structure of risk.

We provide empirical support to the above interpretation in Figure 1. The middle panel of Figure 1 shows the time-series of the logarithm of aggregate dividends and highlights the rare disaster events defined by drops larger than two times the standard deviation of growth rates. Consistent with Gourio (2008), we observe that a substantial recovery occurs during the two years following the rare events.

The lower panel of Figure 1 shows the time-series of aggregate dividends relative to consumption. We observe that the dividend-share moves negatively with disasters and positively with recovery.⁵ Two implications are noteworthy. First, since consumption and dividends are co-integrated, the dividend-share is stationary and cannot depend on the permanent shock. This provides further support to the idea that disasters are transitory shocks. Second, the sensitivity of the dividend-share to disasters and recovery suggests that dividends load more on disaster risk than consumption does. This helps explain why dividends are riskier than

⁵Similar results are obtained if we look at the dividend-share of GDP instead of consumption.

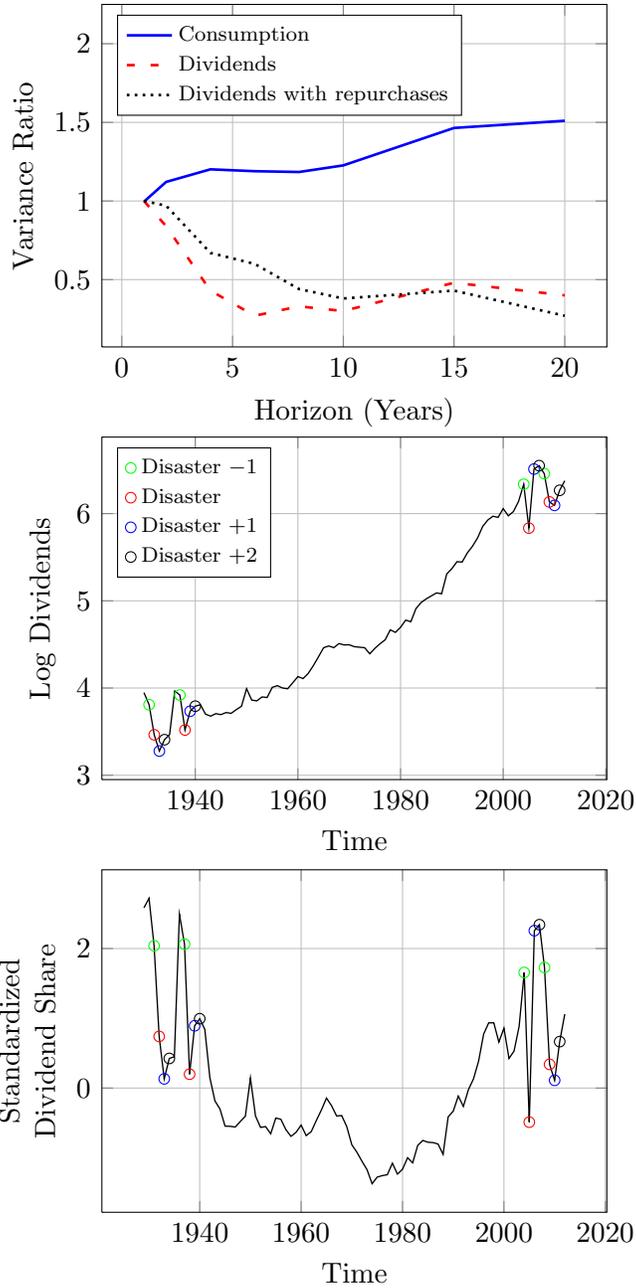


Figure 1: Empirical evidence concerning disasters and recovery in consumption and dividends.

The upper panel displays the variance-ratios (VR) of dividends from the US non-financial corporate sector, as in [Belo, Collin-Dufresne, and Goldstein \(2014\)](#). The VR of the US aggregate consumption are computed using data from [Beeler and Campbell \(2012\)](#). The VR procedure uses the exposition of [Campbell, Lo, and MacKinlay \(1997, pp. 48-55\)](#), which accounts for heteroscedasticity and overlapping observations. The middle and lower panels display the time-series of the logarithm of dividends and the standardized dividend-share of consumption. Red markers denote disasters, i.e., negative yearly growth rates of dividends larger than two standard deviations. Green, blue and black markers denote observations one year before, one year ahead and two years ahead of the disasters, respectively.

consumption at short horizons and why, as commented above, the term-structure of dividend and consumption risk are downward-sloping and upward-sloping, respectively.

Overall, we observe the following stylized facts: i) dividend risk is downward-sloping; ii) consumption risk is upward-sloping; iii) dividends are riskier than consumption at short horizons; iv) dividends and consumption are cointegrated; v) disasters are followed by recovery; vi) dividends load more on disaster risk than consumption does, and vii) the dividend-share moves with disasters and recoveries.

In Sections 3, 4, and 5, we show that recoveries help explain the observed negative slope of the term-structures of equity even if consumption and dividends are equal in equilibrium. In Section 6, we extend the model and assume parsimonious joint dynamics for consumption and dividends, which capture the seven stylized facts mentioned above. This model reconciles standard asset pricing facts, such as the low risk-free rate and the high equity premium, with the downward-sloping term-structure of equity and the pro-cyclical dynamics of its slope.

3 Time-Varying Rare Disasters and Recoveries

In this section, we first describe the economy and then solve for the equilibrium price of dividend strips. In our model, dividends are subject to time-varying rare disasters (Gabaix, 2012; Wachter, 2013) followed by recoveries (Gourio, 2008; Nakamura, Steinsson, Barro, and Ursua, 2013).

3.1 The Economy

We consider a pure-exchange economy à la Lucas (1978) populated by a representative investor with recursive preferences (Epstein and Zin, 1989). The investor's utility function is defined by

$$U_t \equiv \left[(1 - \delta^{dt}) C_t^{\frac{1-\gamma}{\theta}} + \delta^{dt} \mathbb{E}_t (U_{t+dt}^{1-\gamma})^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}},$$

where C is consumption, δ is the subjective discount factor per unit of time, γ is the coefficient of risk aversion, ψ is the elasticity of intertemporal substitution (EIS), and $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$.

The investor can invest in two assets: one stock and one risk-free asset. The stock and the risk-free asset are in positive supply of one unit and in zero net supply, respectively. The

stock pays a continuous stream of dividends, D_t , defined as follows:

$$\begin{aligned}
\log D_t &= x_t + z_t \\
dx_t &= \left(\mu_x - \frac{1}{2}\sigma_x^2\right)dt + \sigma_x dW_{x_t} \\
dz_t &= -\phi z_t dt + \xi_t dN_{z_t} \\
d\lambda_t &= \lambda_r(\lambda_m - \lambda_t)dt + \lambda_v \sqrt{\lambda_t} dW_{\lambda_t},
\end{aligned} \tag{1}$$

where $(W_{x_t}, W_{\lambda_t})^\top$ is a standard Brownian motion and N_t is a pure jump process with stochastic intensity λ_t . The jump size ξ follows a negative exponential distribution with parameter η_t . That is, the jump size is negative and characterized by the following moment-generating function:

$$\varrho(u) \equiv \mathbb{E}_t(e^{u\xi_t}) = \frac{1}{1 + \frac{u}{\eta}}.$$

The log-dividend process is a sum of two terms. The first term, x_t , is the dividend growth rate had there been no disasters and consequently no recoveries either. The aim of the second term, z_t , is to model disasters and recoveries. Conditional on the occurrence of a disaster ($dN_t = 1$), the log-dividend drops instantaneously by an amount ξ_t . Following the drop, the process z_t reverts back to zero at speed ϕ and therefore implies a dividend recovery. If the mean-reversion speed ϕ is equal to zero, there are no recoveries and disasters are permanent.⁶

3.2 Equilibrium

In order to solve for the prices of dividend strips, we follow the procedure documented by Eraker and Shaliastovich (2008), which is based on Campbell and Shiller (1988)'s log-linearization. The first step consists in characterizing the price of the stock, the state-price density and therefore the risk-neutral measure. Then, the price at time t of a dividend strip with residual maturity τ is obtained by computing the expected present value under the risk-neutral measure of a dividend $D_{t+\tau}$ paid at time $t + \tau$.

Recursive preferences lead to a non-affine state-price density. Then, to preserve analytic tractability, we make use of the following log-linearization. The discrete time (continuously compounded) log-return on aggregate wealth (e.g. the claim on the representative investor's

⁶Nakamura, Steinsson, Barro, and Ursua (2013) point out that disasters are not necessarily instantaneous, and can be partly transitory and partly permanent. We keep the model simple and do not account for such patterns because they do not alter the main results of the paper—recoveries imply downward-sloping term-structures of equity.

consumption) can be expressed as

$$\log R_{t+1} = \log \frac{P_{t+1} + D_{t+1}}{P_t} = \log (e^{v_{t+1}} + 1) - v_t + \log \frac{D_{t+1}}{D_t},$$

where $v_t = \log(P_t/D_t)$ (recall $C_t = D_t$ in equilibrium). A log-linearization of the first summand around the mean log price-dividend ratio leads to

$$\log R_{t+1} \approx k_0 + k_1 v_{t+1} - v_t + \log \frac{D_{t+1}}{D_t},$$

where the endogenous constants k_0 and k_1 satisfy

$$k_0 = -\log((1 - k_1)^{1-k_1} k_1^{k_1}) \quad k_1 = e^{\mathbb{E}(v_t)} / (1 + e^{\mathbb{E}(v_t)}).$$

Campbell, Lo, and MacKinlay (1997) and Bansal, Kiku, and Yaron (2012) have documented the high accuracy of such a log-linearization, which we assume exact hereafter. We follow Eraker and Shaliastovich (2008) and consider the continuous time counterpart defined in the following way:

$$d \log R_t = k_0 dt + k_1 dv_t - (1 - k_1)v_t dt + d \log D_t.$$

Recursive preferences lead to the following Euler equation, which enables us to characterize the state-price density, M , used to price any asset in the economy:

$$\mathbb{E}_t \left[\exp \left(\log \frac{M_{t+\tau}}{M_t} + \int_t^{t+\tau} d \log R_s \right) \right] = 1.$$

The state-price density satisfies

$$d \log M_t = \theta \log \delta dt - \frac{\theta}{\psi} d \log D_t - (1 - \theta) d \log R_t.$$

To solve for the return on aggregate wealth and, in turn, on the state-price density, one has to conjecture that v_t is affine in the state-vector $Y_t = (x_t, z_t, \lambda_t)^\top$. Then, the above Euler equation is used to solve for the coefficients. The price of the stock, which equals aggregate wealth, is characterized in Proposition 1 below.

Proposition 1. *In equilibrium, the investor consumes the dividend paid by the stock. Therefore, the investor's wealth is equal to the stock price, P_t , which satisfies*

$$v_t \equiv \log \frac{P_t}{D_t} = A + B^\top Y_t,$$

where “ \top ” is the transpose operator, and $Y_t = (x_t, z_t, \lambda_t)^\top$ is the vector of state variables.

The state variables belong to the affine class and their dynamics can be written as:

$$\begin{aligned}
dY_t &= \mu(Y_t)dt + \Sigma(Y_t)dW_t + J_t \cdot dN_t \\
\mu(Y_t) &= \mathcal{M} + \mathcal{K}Y_t \\
\Sigma(Y_t)\Sigma(Y_t)^\top &= h + \sum_{i=1}^3 H^i Y_t^i \\
l(Y_t) &= l_0 + l_1 Y_t,
\end{aligned}$$

where $\mathcal{M} \in \mathbb{R}^3$, $\mathcal{K} \in \mathbb{R}^{3 \times 3}$, $h \in \mathbb{R}^{3 \times 3}$, $H \in \mathbb{R}^{3 \times 3 \times 3}$, $l_0 \in \mathbb{R}^3$, and $l_1 \in \mathbb{R}^{3 \times 3}$, $W = (W_x, W_z, W_\lambda)^\top$ is a standard Brownian motion; $N_t = (N_{xt}, N_{zt}, N_{\lambda t})^\top$ is a vector of independent pure jump processes; $l(Y) \in \mathbb{R}^3$ is the corresponding vector of jump intensities; $J_t \in \mathbb{R}^3$ is the vector of jump sizes; and “ \cdot ” denotes element-by-element multiplication.

The coefficients $A \in \mathbb{R}$ and $B \in \mathbb{R}^3$ solve a system of equations provided in Appendix A.1.

Proof. See Appendix A.1.

The price-dividend ratio is stationary in our model and consequently independent of x_t ($B_1 = 0$). Moreover, the price-dividend ratio decreases (resp., increases) with the intensity λ_t when the intertemporal elasticity of substitution is larger (resp., smaller) than one. The reason is as follows. An increase in the intensity increases the likelihood that negative jumps in dividends will take place in the future. This has two opposite effects on the investor’s behavior and therefore on prices. First, the possibility that more jumps might occur implies that the investor decreases his stock holdings. This *investment effect* generates a drop in the price. Second, the possibility of more frequent downward jumps implies lower future consumption. Because the investor wants to smooth consumption over time, he decreases current consumption and hence invests more in the stock. This *consumption effect* increases the stock price. The investment effect dominates when the intertemporal elasticity of substitution is larger than one, whereas the consumption effect is the strongest when the elasticity is smaller than one.

For a similar reason, the price-dividend ratio decreases (resp., increases) with the jump process z_t when the EIS is larger (resp., smaller) than one. Indeed, a drop in z_t , today, indicates that future dividend growth will be large because a recovery will take place. Because the investment effect outweighs the consumption effect when the EIS is larger than one, a drop in z_t implies an increase in the stock price. Conversely, a drop in z implies a decrease in the stock price when the EIS is smaller than one because, in this case, the consumption effect dominates.

The risk-free rate r_t , market price of continuous risk vector Λ_t^c , and market price of jump risk vector Λ_t^d are defined in Proposition 2.

Proposition 2. *The dynamics of the state-price density M are written*

$$\frac{dM_t}{M_t} = -r_t dt - \Lambda_t^c dW_t - \sum_{i=1}^3 \left(\Lambda_t^{d^i} dN_t^i - \mathbb{E}_t \left(\Lambda_t^{d^i} \right) l^i(Y_t) dt \right),$$

where the risk-free rate r_t , the market price of continuous risk Λ_t^c , and the market price of jump risk Λ_t^d satisfy

$$\begin{aligned} r_t &= \Phi_0 + \Phi_1^\top Y_t, \\ \Lambda_t^c &= \Sigma(Y_t)^\top \Omega, \\ \Lambda_t^{d^i} &= 1 - e^{\Omega^i J_t^i}, \quad \forall i. \end{aligned}$$

The coefficient $\Phi_0 \in \mathbb{R}$ and $\Phi_1 \in \mathbb{R}^3$ solve a system of equations provided in Appendix A.2.

Proof. See Appendix A.2.

The risk-free rate is stationary and therefore independent of x_t . It is, however, a decreasing linear function of the intensity λ_t , irrespective of the value assigned to the EIS. The reason is that an increase in the expected frequency of a disaster increases the representative investor's desire to save, and thus decreases the risk-free rate level. Such an effect increases in magnitude with relative risk aversion. The decreasing linear relation between the risk-free rate and the disaster process z_t is understood as follows. A disaster, i.e. a drop in z_t , will imply a recovery and, therefore, high future dividend growth. On the one hand, because future investment opportunities improve, the investor reduces risk-free asset holdings to increase risky investments if EIS is larger than one. On the other hand, larger future consumption implies an increase in current consumption and therefore a decrease in both risky and risk-free position if EIS is lower than one. In both cases, risk-free holdings move positively with z_t , indicating that the risk-free rate decreases with it.

The first component of the market price of continuous risk Λ_t^c is constant and equal to $\gamma\sigma_x$ (Lucas, 1978): it rewards the investor for bearing the constant dividend growth risk. Another component of Λ_t^c is associated with jump intensity. Such a term is stochastic because the jump intensity features a square-root diffusion (see Equation (1)). Moreover, the investor gets a reward Λ_t^d for bearing the risk of a jump in z_t : such a term is constant because the jump size ξ_t features an i.i.d. distribution.

Proposition 3 characterizes the price of a dividend strip with time to maturity τ .

Proposition 3. *The price at time t of a dividend strip with time to maturity τ is written*

$$\begin{aligned} S_t(\tau) &= \mathbb{E}_t^{\mathbb{Q}} \left(e^{-\int_t^{t+\tau} r_s ds} D_{t+\tau} \right) \\ &= e^{a(\tau)+b(\tau)^\top Y_t}, \end{aligned}$$

where \mathbb{Q} is the risk-neutral measure implicitly defined by Proposition 2.

The deterministic functions $a(\cdot) \in \mathbb{R}$ and $b(\cdot) \in \mathbb{R}^3$ solve a system of ordinary differential equations provided in Appendix A.3.

Proof. See Appendix A.3.

Then, Itô's lemma and Girsanov's theorem yield the volatility and the risk premium of the dividend strip provided in Proposition 4.

Proposition 4. *The variance of a dividend strip with time to maturity τ is written*

$$\|\sigma_t(\tau)\|^2 = \|\sigma_t^c(\tau)\|^2 + \|\sigma_t^d(\tau)\|^2,$$

where $\|\sigma_t^c(\tau)\|^2$ and $\|\sigma_t^d(\tau)\|^2$ are the variances implied by the Brownian motion W_t and the jump process N_t , respectively. The vectors $\sigma_t^c(\tau)$ and $\sigma_t^d(\tau)$ satisfy

$$\begin{aligned} \sigma_t^c(\tau) &= \frac{1}{S_t(\tau)} \left(\frac{\partial}{\partial Y} S_t(\tau) \right)^\top \Sigma(Y_t) \\ \sigma_t^d(\tau) &= \sqrt{l(Y_t)^i \mathbb{E} \left[(e^{b^i(\tau)J_t^i} - 1)^2 \right]} = \sqrt{l(Y_t)^i (\varrho(2b^i(\tau)) - 2\varrho(b^i(\tau)) + 1)}. \end{aligned}$$

The risk premium on a dividend strip with time to maturity τ is written

$$RP_t(\tau) = RP_t^c(\tau) + RP_t^d(\tau),$$

where $RP_t^c(\tau)$ and $RP_t^d(\tau)$ are the premiums for bearing Brownian and jump risks, respectively. These risk premia satisfy

$$\begin{aligned} RP_t^c(\tau) &= \sigma_t^c(\tau) \Lambda_t^c \\ RP_t^d(\tau) &= \sum_{i=1}^3 l(Y_t)^i \mathbb{E} \left[\left(e^{b^i(\tau)J_t^i} - 1 \right) \Lambda_t^{d^i} \right] = \sum_{i=1}^3 l(Y_t)^i (\varrho(b^i(\tau)) - \varrho(b^i(\tau) - \Omega^i) + \varrho(-\Omega^i) - 1). \end{aligned}$$

Proof. Straightforward application of Itô's lemma and Girsanov's theorem.

The level and shape of the term-structures of volatility and risk premia are inherited by the level and shape of the term-structure of dividend growth rate volatility when EIS is larger than one. We start with a discussion on the term-structure of dividend growth rate volatility in Section 4 and then document the properties of the term-structures of return volatility and risk premia in Section 5.

4 Term Structure of Dividend Risk

In this section, we discuss the term-structure of dividend growth rate volatility because it is the main determinant of the shape of the term-structures of equity volatility and risk premium.

If not mentioned otherwise, the values of the parameters used in our analysis are presented in Table 1. The parameters are borrowed from Wachter (2013) with the exception of recovery from disaster—our peculiar parameter.⁷ We set a relatively small speed of recovery ($\phi = 7.5\%$) such that, after a drop, it takes the dividend about five years to recover. This is due to the joint effect of the mean-reversion of z_t and the positive expected growth of x_t , and is consistent with the international evidence documented by Gourio (2008). The jump size parameter implies that, conditional to a jump, the dividend drops by 20% on average (Barro, 2006; Barro and Ursua, 2008). If not mentioned otherwise, the state variables are set at their steady-state ($\lambda_t = \lambda_m, z_t = 0$), whereas the value of x_t is irrelevant since the equilibrium is stationary.

The term-structure of dividend volatility and variance ratio is characterized in Proposition 5 below.

Proposition 5. *The dividend growth rate volatility at a τ -year horizon, $\sigma_D(t, \tau)$, is written*

$$\sigma_D(t, \tau) = \sqrt{\frac{1}{\tau} \log \left(\frac{MGF(t, \tau; 2)}{MGF(t, \tau; 1)^2} \right)},$$

where the moment-generating function $MGF(., .)$ satisfies

$$MGF(t, \tau; u) = \mathbb{E}_t \left(D_{t+\tau}^u \right) = e^{\bar{a}(\tau; u) + \bar{b}(\tau; u)^\top Y_t}.$$

The deterministic functions $\bar{a}(., .) \in \mathbb{R}$ and $\bar{b}(., .) \in \mathbb{R}^3$ solve a system of ordinary differential equations provided in Appendix A.4.

⁷Parameters $\sigma_x, \mu_x, \lambda_m, \lambda_r$ and λ_v are as in Wachter (2013), jump size distribution is not exactly the same but $\eta = 4$ captures the same average jump size.

Parameter	Symbol	Value
Permanent shock:		
Instantaneous volatility	σ_x	0.02
Long-run expected growth	μ_x	0.0252
Disasters:		
Speed of reversion of jump intensity	λ_r	0.08
Long-term jump intensity	λ_m	0.0355
Volatility of jump intensity	λ_v	0.067
Jump size parameter	η	4
Speed of recovery	ϕ	0.075
Preferences:		
Relative risk aversion	γ	3
Elasticity of intertemporal substitution (EIS)	ψ	1.5
Time discount factor	δ	0.96

Table 1: Calibration

The dividend variance ratio at a τ -year horizon, $VR_D(t, \tau)$, is defined as follows:

$$VR_D(t, \tau) = \frac{\sigma_D(t, \tau)^2}{\sigma_D(t, 1)^2}.$$

Proof. See Appendix A.4.

Notice that $\sigma_D(t, \tau)$ captures both the continuous and the discontinuous expected variation in dividends. Moreover, the first component of $\bar{b}(\tau; u)$ is simply u , such that $\sigma_D(t, \tau)$ and $VR_D(t, \tau)$ do not depend on x_t .

Figure 2 shows that the term-structures of dividend volatility and variance ratio are decreasing when dividends recover and increasing when they do not. The larger the speed of recovery is, the larger in magnitude the negative slope of these term-structures become. That is, accounting for recoveries helps explain the observed shape of the term-structures of dividend risk and variance ratio.⁸ The reason is as follows. The two main components of dividend risk are the disaster risk and the intensity risk.⁹ Time variation in jump intensity, λ_t , induces an upward-sloping effect on the term-structure because it induces uncertainty

⁸Beeler and Campbell (2012), Belo, Collin-Dufresne, and Goldstein (2014), and Marfè (2014) provide empirical evidence of a decreasing term-structure of dividend variance ratios.

⁹Note that we do not discuss the risk implied by the Brownian motion W_x because it is constant over time.

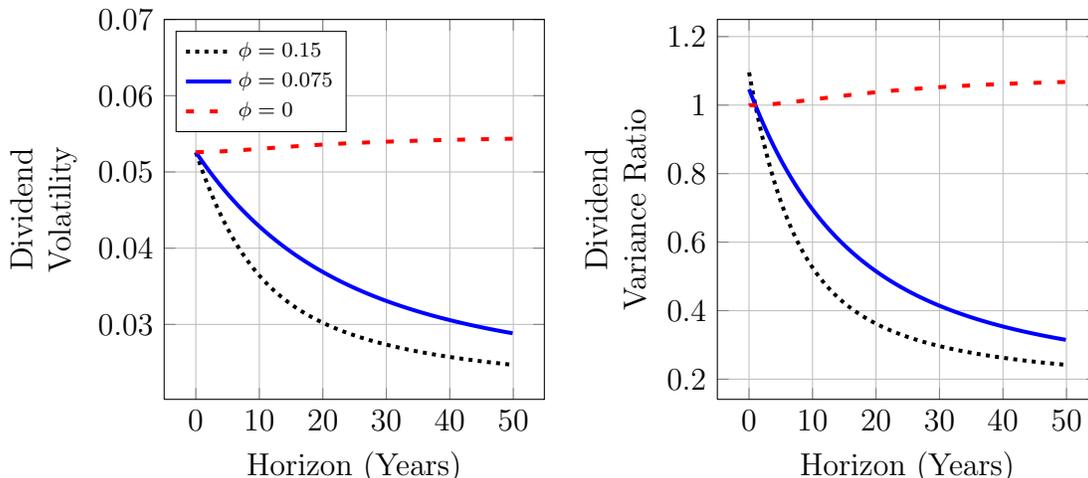


Figure 2: Term structure of dividend volatility and variance ratio vs. speed of recovery.

The left panel depicts the volatility of dividends over horizons ranging from 0 to 50 years. The right panel depicts the corresponding dividends variance ratio. That is, the τ -year dividends variance relative to the 1-year dividends variance. The calibration is provided in Table 1.

concerning the frequency of disaster risk. Consider now the role of z_t . Over short horizons (e.g., 1 day), recoveries do not influence disaster risk. Over long horizons (e.g., 20 years), however, recoveries dampen the disaster risk that would have prevailed, had there been no recoveries ($\phi = 0$). Therefore, recovery from disasters induces a downward-sloping effect on the term-structure. Overall, the downward-sloping term-structure of recovery outweighs the upward-sloping term-structure effect of time-varying intensity. This generates a downward-sloping term-structure of dividend risk and variance ratio when there are recoveries ($\phi > 0$).

We assume from now on that the disaster intensity is a proxy for economic conditions. That is, good, normal, and bad times correspond to a low, moderate, and high disaster intensity, respectively. Figure 3 shows that dividend volatility and variance ratio decrease with the time horizon in bad and normal times, whereas they increase with it in good times. The reason is as follows. In good times, the disaster intensity is expected to revert back up to its long-term mean, generating more risk in the long-term than in the short-term. In normal times, the disaster intensity is expected remain steady. As mentioned earlier, the fact that dividends are expected to recover in the long term implies less risk in the long term than in the short term. In bad times, the disaster intensity is expected to revert back down to its long-term mean, further reducing the risk of long-term dividends.

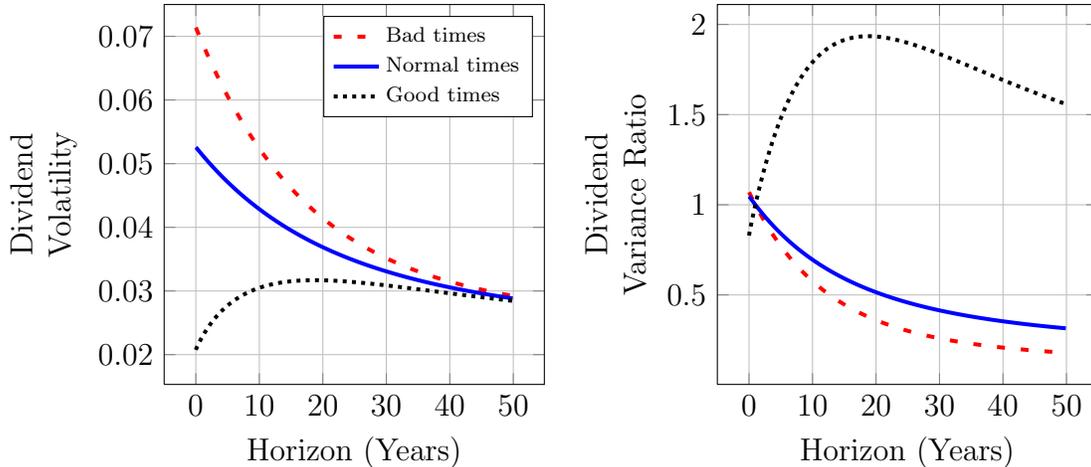


Figure 3: Term structure of dividend volatility and variance ratio vs. economic conditions.

The left panel depicts the volatility of dividends over horizons ranging from 0 to 50 years. The right panel depicts the corresponding dividends variance ratio. That is, the τ -year consumption variance relative to the 1-year dividends variance. Good, normal, and bad times correspond to a jump intensity $\lambda = \lambda_m - 0.035$, $\lambda = \lambda_m$, and $\lambda = \lambda_m + 0.035$, respectively. The calibration is provided in Table 1.

5 Term Structure of Equity Volatility and Risk Premium

In this section we discuss the properties of the term-structures of equity volatility and risk premia. In particular, we show that those are increasing when the risk of disaster is time-varying and decreasing when the model is extended to account for recoveries. That is, recoveries help reconcile the literature on time-varying risk of disasters (Gabaix, 2012; Wachter, 2013) and the empirical literature documenting downward-sloping term-structures of equity volatility and risk premia (van Binsbergen, Brandt, and Koijen, 2012; van Binsbergen, Hueskes, Koijen, and Vrugt, 2013).

We complete the model calibration by selecting the preference parameters. Namely, we set the relative risk aversion and the time discount factor to standard values ($\gamma = 3$, $\delta = .96$). Then, we follow Bansal and Yaron (2004) and choose an elasticity of intertemporal substitution (EIS) larger than one ($\psi = 1.5$). In equilibrium, this condition implies a relatively low risk-free rate volatility.

Figure 4 shows that the term-structures of equity volatility and risk premia inherit the properties of the term-structure of dividend risk when $EIS > 1$. Indeed, when the elasticity of intertemporal substitution is larger than one, these term-structures are decreasing when accounting for recoveries and increasing when they do not. Moreover, the negative slopes

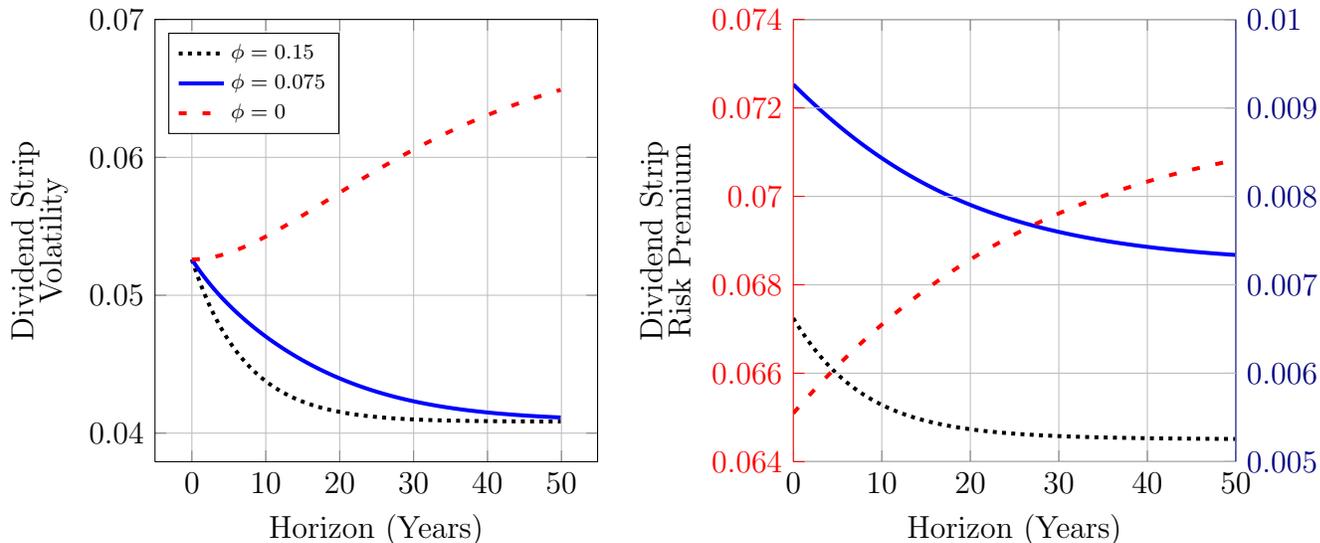


Figure 4: Term structure of equity volatility and risk premia vs. speed of recovery.

The left panel depicts the volatility of dividend strips for horizons ranging from 0 to 50 years. The right panel depicts the term-structure of risk premia. The red left-axis describes the red curve. The dark blue right-axis describes the blue and black curves. The calibration is provided in Table 1.

of the term-structures increase in magnitude with the speed of recovery. The reason is that time is needed for the dividend to recover. Therefore, short-term dividends face the risk of a disaster and the benefit of a short recovery, whereas long-term dividends benefit from a long recovery period. Because the dividend effect is stronger than the discounting effect when the EIS is larger than one, returns become more risky in the short-term than in the long-term, i.e., equity volatility and risk premia are larger in the short-term than in the long-term. It is worth noting that, even though recoveries help explain the shapes of the term-structures of equity, recoveries significantly decrease the level of the risk premium. We show in Section 6 that this trade off can be relaxed, and this empirical inconsistency can be resolved by accounting for the joint dynamics of consumption and dividends and their co-integration.

Consistent with the empirical findings of van Binsbergen, Hueskes, Koijen, and Vrugt (2013), Figure 5 shows that the slopes of the term-structures of equity volatility and risk premia become negative as economic conditions deteriorate. Indeed, the term-structures of equity are flat in good times and decreasing in both normal and bad times. In normal and bad times, returns are riskier in the short term than in the long term because of the existence of recoveries. In good times, however, the disaster intensity is expected to revert back up to its long-term mean. This tends to increase the risk of long-term dividends and therefore dampens the downward-sloping effect of recoveries.

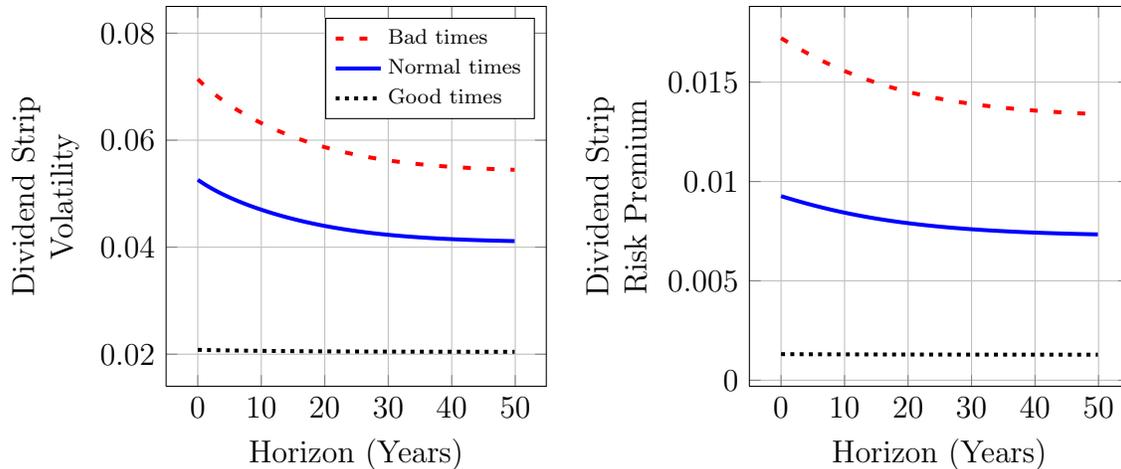


Figure 5: Term structure of equity volatility and risk premia vs. economic conditions.

The left panel depicts the volatility of dividend strips for horizons ranging from 0 to 50 years. The right panel depicts the term-structure of risk premia. Good, normal, and bad times correspond to a jump intensity $\lambda = \lambda_m - 0.035$, $\lambda = \lambda_m$, and $\lambda = \lambda_m + 0.035$, respectively. The calibration is provided in Table 1.

Figure 6 shows that the downward-sloping effect of recoveries on the term-structures of equity only holds for specific values of the elasticity of intertemporal substitution. Indeed, the term-structures of equity are flat if the representative investor is myopic ($EIS = 1$), upward-sloping if he has CRRA preferences, and downward-sloping if the EIS is larger than one. When the investor is myopic, price-dividend ratios are constant in equilibrium. Consequently, the risk of short-term assets is the same as the risk of long-term assets. When the investor has CRRA preferences, the discounting effect is stronger than the dividend effect. As a result, returns inherit the upward-sloping properties of discount rates instead of the downward-sloping properties of dividend growth rates. When EIS is larger than one, however, the opposite holds and the term-structures of equity are downward-sloping.

6 Extension: a Quantitative Solution of the Equity Premium Puzzle

In this section we still consider an endowment economy with a representative agent having recursive preferences. However, we assume parsimonious joint dynamics for aggregate consumption and dividends, which capture the empirical patterns documented in Section 2.

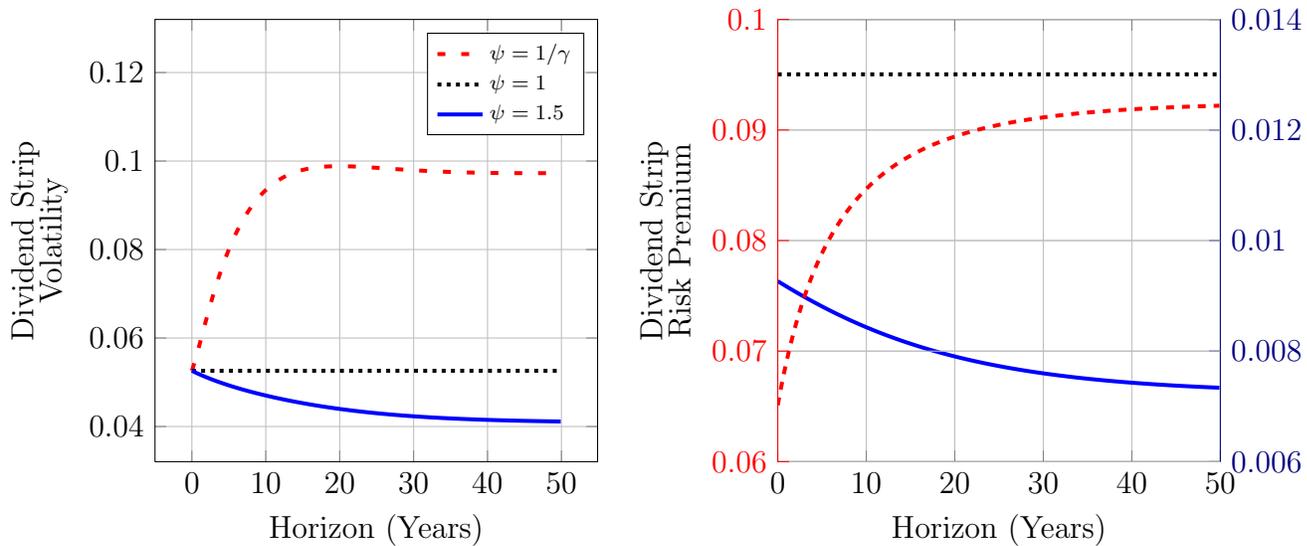


Figure 6: Term structure of equity volatility and risk premia vs. elasticity of intertemporal substitution.

The left panel depicts the volatility of dividend strips for horizons ranging from 0 to 50 years. The right panel depicts the term-structure of risk premia. The red left-axis describes the red curve. The dark blue right-axis describes the blue and black curves. The calibration is provided in Table 1.

Consumption and dividends are characterized as follows:

$$\begin{aligned} \log C_t &= x_t + z_t, \\ \log D_t &= x_t + \alpha z_t + \log d_0, \quad \alpha \geq 1, d_0 \in (0, 1), \end{aligned}$$

with

$$\begin{aligned} dx_t &= (\mu_t - \frac{1}{2}\sigma_x^2)dt + \sigma_x dW_{xt}, \\ dz_t &= -\phi z_t dt + \xi_t dN_{zt}, \\ d\lambda_t &= \lambda_r(\lambda_m - \lambda_t)dt + \lambda_v \sqrt{\lambda_t} dW_{\lambda t}, \\ d\mu_t &= \kappa(\bar{\mu} - \mu_t)dt + \nu dW_{\mu t}. \end{aligned}$$

Notice that $z_t \leq 0$, hence $0 < D_t \leq C_t$ and the dividend-share D_t/C_t is properly defined. Moreover, the model of the previous sections is obtained as a special case for $\alpha = d_0 = 1$ and $\nu \rightarrow 0$.

Because the permanent shock x_t enters the equation for both $\log C_t$ and $\log D_t$ with the same coefficient, consumption and dividends are co-integrated in levels, as observed in the data. In order to generate an upward-sloping term-structure of consumption growth volatility,

Parameter	Symbol	Value
Permanent shock:		
Instantaneous volatility	σ_x	0.02
Long-run expected growth	μ_x	0.0252
Volatility of expected growth	ν	0.02
Speed of reversion	κ	0.25
Disasters:		
Speed of reversion of jump intensity	λ_r	0.08
Long-term jump intensity	λ_m	0.0355
Volatility of jump intensity	λ_v	0.067
Jump size parameter	η	4
Speed of recovery	ϕ	0.075
Dividends:		
Leverage parameter	α	3
Steady state dividend-share	d_0	0.05
Preferences:		
Relative risk aversion	γ	5
Elasticity of intertemporal substitution (EIS)	ψ	{2/3, 1, 3/2}
Time discount factor	δ	0.96

Table 2: Calibration

we add to the model time-variation in expected growth μ_t (Bansal and Yaron, 2004). As in the previous sections, a disaster is followed by a recovery ($\phi > 0$). In addition, we embed a levered exposition of dividends to disaster risk ($\alpha > 1$). The latter assumption produces the following properties: i) although dividends and consumption are co-integrated, dividends are more volatile than consumption at short horizons; ii) the term-structure of dividend risk is downward-sloping because $\alpha > 1$ allows the downward-sloping effect implied by recoveries to outweigh the upward-sloping effect implied by long-run risk and time-varying intensity; and iii) the dividend-share $D_t/C_t = d_0 e^{(\alpha-1)z_t}$ varies with disasters and recoveries.

We calibrate the model as follows. First, we preserve all parameter values from Section 4. Second, we set the additional parameters concerning expected growth, that is $\kappa = 0.25$ and $\nu = 1.25\%$, in the range of values considered in the long-run risk literature. Third, we set $d_0 = 5\%$ to approximately match the unconditional level of the dividend-share of consumption in the U.S. non-financial corporate sector. Fourth, we set the excess disaster risk of dividends at $\alpha = 3$ as in Wachter (2013).

Figure 7 displays the term-structures of dividends volatility (on the left) and the corresponding variance-ratios (on the right) for several values of α . In line with empirical findings

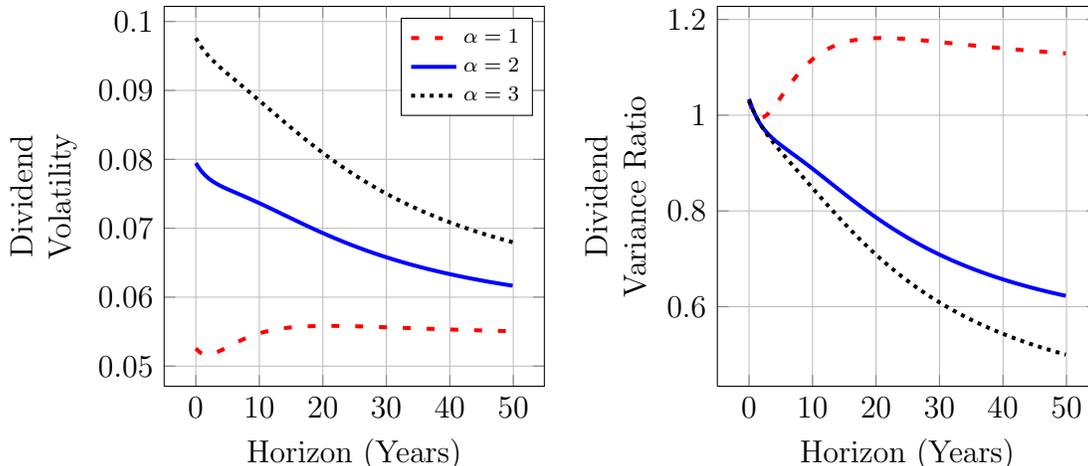


Figure 7: Term-structure of volatility and variance ratio for consumption and dividends.

The left panel depicts the volatility of consumption and dividends over horizons ranging from 0 to 50 years. The right panel depicts the corresponding variance ratios. That is, the τ -year variance relative to the 1-year variance. Consumption is obtained for leverage parameter α equal to one. Dividends are levered consumption and are obtained for $\alpha \geq 1$. The calibration is provided in Table 2.

(see Figure 1), the model implies strongly downward-sloping and slightly upward-sloping term-structures of dividend ($\alpha = 3$) volatility and consumption ($\alpha = 1$) volatility, respectively. However, the model somewhat underestimates the level of short-run dividend risk (about 9%) and overestimates the level of consumption risk (about 5%). This can be fixed by adding an idiosyncratic and transitory shock to dividends. For the sake of simplicity and because it would not affect the model implications on the term-structure of equity, we do not add such a shock.¹⁰

Since the above joint dynamics for consumption and dividends belongs to the affine class, we can collect the model state-variables in an affine vector, $Y_t = (x_t, z_t, \lambda_t, \mu_t)$, and solve for the equilibrium as in the previous sections by following Eraker and Shaliastovich (2008). The aggregate wealth (i.e. the claim on aggregate consumption) is such that the wealth-consumption ratio is stationary and exponential affine with constant coefficients:

$$v_{c,t} \equiv \log \frac{W_t}{C_t} = A_c + B_c^\top Y_t.$$

¹⁰An idiosyncratic and transitory shock to dividends would also smooth the dynamics of the dividend-share, which only varies with disasters and recoveries in our setup.

The state-price density satisfies

$$\frac{dM_t}{M_t} = -r_t dt - \Lambda_t^c dW_t - \sum_{i=1}^4 \left(\Lambda_t^{di} dN_t^i - \mathbb{E}_t \left(\Lambda_t^{di} \right) l^i(Y_t) dt \right),$$

where the risk-free rate r_t is affine in Y_t and the market price of (continuous) risk Λ_t^c includes an additional term associated to time-variation in expected growth. The latter is positive (resp. negative) under preference for early (resp. late) resolution of uncertainty (Bansal and Yaron, 2004).

The prices of the strips which pay either consumption or dividends at maturity are exponential affine

$$\begin{aligned} S_t^{\text{con}}(\tau) &= \mathbb{E}_t^{\mathbb{Q}} \left(e^{-\int_t^{t+\tau} r_s ds} C_{t+\tau} \right) = e^{a_c(\tau) + b_c(\tau)^\top Y_t}, \\ S_t^{\text{div}}(\tau) &= \mathbb{E}_t^{\mathbb{Q}} \left(e^{-\int_t^{t+\tau} r_s ds} D_{t+\tau} \right) = e^{a_d(\tau) + b_d(\tau)^\top Y_t}, \end{aligned}$$

and equal each other for $\alpha = 1$. The valuation-ratios $S_t^{\text{con}}(\tau)/C_t$ and $S_t^{\text{div}}(\tau)/D_t$ do not depend on x_t and, hence, are stationary. Similar to Proposition 4, an application of Itô's lemma to the dividend strip price, $S_t^{\text{div}}(\tau)$, allows us to derive the term-structures of both equity risk, $\|\sigma_t(\tau)\|$, and equity premia $RP_t(\tau)$.

Finally, the price of equity (i.e. the claim on aggregate dividends) can be either computed by numerical integration of the dividend strip prices over the time horizon

$$P_t = \int_0^\infty S_t^{\text{div}}(\tau) d\tau,$$

or approximated through a log-linearization of the return process. In that case, the price satisfies the following exponential affine form: $P_t \approx D_t e^{A_d + B_d^\top Y_t}$. The latter expression is particularly convenient when pricing other assets such as options (see Eraker and Shaliastovich (2008)).

In what follows, we first analyze the model predictions concerning dividend strips. Then, we show how the model reconciles a solution of the risk-free rate and equity premium puzzle with an adequate description of the term-structures of consumption risk, dividend risk, and equity.

In order to understand the joint impact of disasters/recoveries and the investor's preference for the timing of risk, we investigate the cases in which the elasticity of intertemporal substitution is equal to $\frac{2}{3}$, 1 and $\frac{3}{2}$. Relative risk aversion is set to a moderate value of 5, which is sufficient to generate sizeable premia on equity. Finally, the time-discount factor is set to $\delta = 0.96$ to control the level of the risk-free rate.

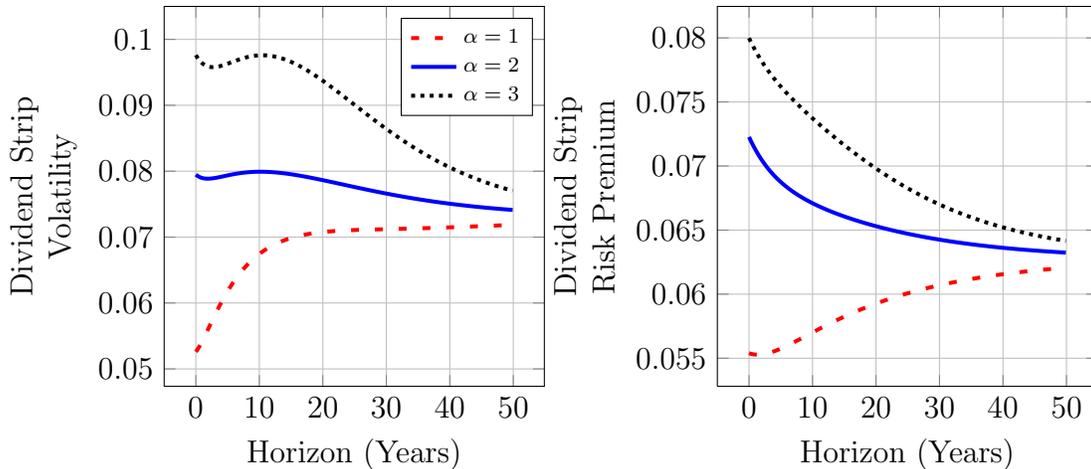


Figure 8: Term-structure of equity volatility and risk premia vs. leverage parameter.

The left panel depicts the volatility of dividend strips for horizons ranging from 0 to 50 years. The right panel depicts the term-structure of risk premia. The red dashed curve, the blue solid curve and the black dotted curve denote the cases $\alpha = 1$, $\alpha = 2$ and $\alpha = 3$, respectively. The calibration is provided in Table 2.

Figure 8 displays the term-structures of equity volatility (left) and equity risk premia (right) for several values of the leverage parameter α . The preference parameters are $\psi = \frac{2}{3}$, $\gamma = 5$.

When the elasticity of intertemporal substitution is lower than one, the consumption effect dominates the investment effect and the term-structures associated with the strip on consumption ($\alpha = 1$) are upward-sloping. However, when dividends load more than consumption on disaster risk ($\alpha > 1$), a moderate investment effect (due to $\psi < 1$) is sufficient to generate downward-sloping term-structures of dividend strips. Indeed, even if the desire to invest in equity is not large (i.e., the representative agent does not want to postpone consumption), equity compensation is high in the short-term because dividends are highly sensitive to disaster risk ($\alpha > 1$), and disaster risk concentrates at short horizons in the presence of recovery ($\phi > 0$). The larger α is, the smaller the elasticity of intertemporal substitution needed to switch the sign of the slope on the term-structures of dividend strips.

Let us now fix the leverage effect at $\alpha = 3$ and look at the role of the elasticity of intertemporal substitution in Figure 9. We observe two noteworthy results. First, equity risk equals dividend risk at very short horizons ($\tau \rightarrow 0$) and inherits the negative slope of dividends when α is large enough. However, the larger the elasticity of intertemporal substitution, the steeper the negative slope of equity risk becomes. Then, a moderate elasticity of intertemporal substitution (e.g. $\psi = \frac{2}{3}$) helps to mitigate the excess volatility of dividends relative to equity at long horizons—an implication of existing models which is inconsistent with the

data (Beeler and Campbell, 2012).

Second, the slope of equity premia is negative when α is large enough, but the level of risk premia is decreasing with the elasticity of intertemporal substitution. For $\psi = \frac{2}{3}$, risk premia lie between 6% and 8% over the horizon between zero and 50 years. As recognized by Gourio (2008), required returns for bearing disaster risk are decreasing with ψ in the presence of recovery. The reason is twofold. First, recoveries imply a decreasing term-structure of dividend risk, meaning that risk is concentrated in the short-term. Second, the representative agent requires a low (resp. high) return for bearing short-term risk when he has a preference for early (resp. late) resolution of uncertainty. Since an increase in the elasticity of intertemporal substitution is synonymous to a preference for earlier resolution of uncertainty, the relation between risk premia and the parameter ψ is decreasing. The opposite holds when the term-structure of dividend risk is upward-sloping, i.e., when there is no recovery. In this case, risk premia increase with ψ because the larger the elasticity of intertemporal substitution, the larger the required compensation for bearing long-term risk.

This reasoning explains why previous disaster models (Wachter, 2013) as well as long-run risk models (Bansal and Yaron, 2004) have difficulty reconciling downward-sloping term-structures with a high equity premium. Indeed, both classes of models assume cash-flow dynamics that imply an upward-sloping term structure of dividend risk. Therefore, these models can either generate high risk premia and upward-sloping term-structures of equity if the agent has a preference for early resolution of uncertainty ($\gamma > 1/\psi$) or low risk premia and downward-sloping term-structures of equity if the agent has a preference for late resolution of uncertainty ($\gamma < 1/\psi$).

Our setup mitigates this tension by improving on the timing of consumption risk and dividend risk. Even for relatively low preference parameters ($\gamma = 5, \psi = \frac{2}{3}$), the transitory nature of disasters ($\phi > 0$) and the larger sensitivity of dividends relative to consumption on disaster risk ($\alpha > 1$) are sufficient to generate high compensations and downward-sloping term structures of equity.

Figure 10 considers the role of time-varying intensity of disasters. Similar to our benchmark model where consumption equals dividends, the downward-sloping term-structures of equity risk and equity premia steepen when economic conditions switch from good times (low λ_t) to bad times (high λ_t). This occurs because, in the presence of recovery, short-term dividend risk increases with λ_t , whereas long-term dividend risk is essentially unaffected. As shown by van Binsbergen, Hueskes, Kojien, and Vrugt (2013), this result finds strong empirical support.

To complete the analysis of the model implications, we look at the levels of the risk-free rate and the equity premium. Table 3 shows how these variables vary for several pairs of

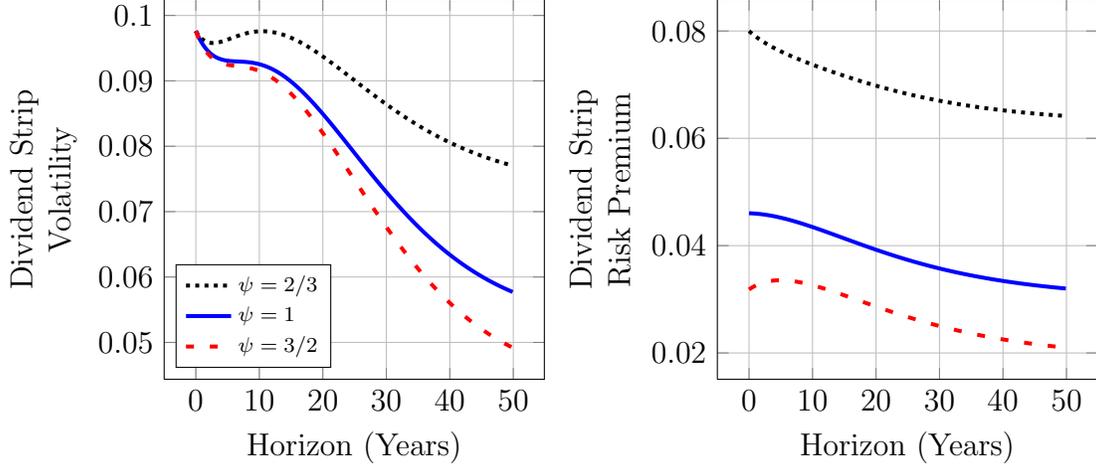


Figure 9: Term-structure of equity volatility and risk premia vs. elasticity of intertemporal substitution.

The left panel depicts the volatility of dividend strips for horizons ranging from 0 to 50 years. The right panel depicts the term-structure of risk premia. The black dotted curve, the blue solid curve and the red dashed curve denote the cases $\psi = 2/3$, $\psi = 1$ and $\psi = 3/2$, respectively. The calibration is provided in Table 2.

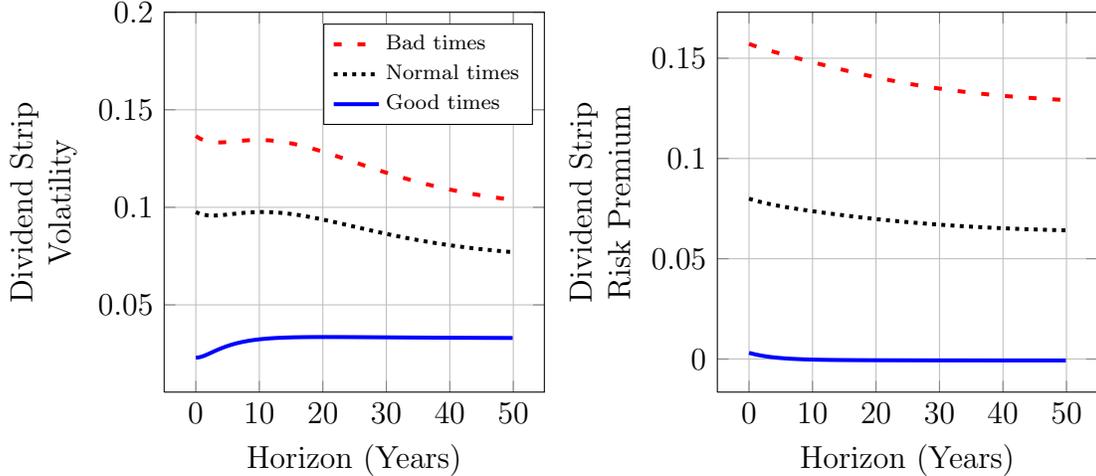


Figure 10: Term-structure of equity volatility and risk premia vs. economic conditions.

The left panel depicts the volatility of dividend strips for horizons ranging from 0 to 50 years. The right panel depicts the term-structure of risk premia. Good, normal, and bad times correspond to a jump intensity $\lambda = \lambda_m - 0.035$, $\lambda = \lambda_m$, and $\lambda = \lambda_m + 0.035$, respectively. The calibration is provided in Table 2.

preference parameters γ and ψ . The equity premium increases with relative risk aversion and decreases with the elasticity of intertemporal substitution. As noted above, the latter result comes from the fact that the main source of equity premium is disaster risk, which is transitory in the presence of recovery ($\phi > 0$). We observe that the pair $\gamma = 5$ and $\psi = \frac{2}{3}$

%	$\psi = 2/3$		$\psi = 1$		$\psi = 3/2$	
	r	RP	r	RP	r	RP
$\gamma = 2$	5.3	1.8	5.0	1.1	4.7	0.8
$\gamma = 5$	0.6	7.2	2.9	4.0	3.5	2.8
$\gamma = 7.5$	-37.2	48.6	-6.1	16.5	-0.7	9.9

Table 3: Risk-free rate and equity premium vs. preference parameters: comparative statics.

leads to a sizeable and realistic equity premium (about 7.2%) and a low risk-free rate (about 0.6%). Therefore, the model reconciles the risk-free rate and equity premium puzzle with the recent empirical evidence concerning the term-structures of equity (van Binsbergen, Brandt, and Koijen, 2012).¹¹ The levered exposition of dividends to disaster risk is a property of the data (see Section 2), which helps obtain downward-sloping term-structures of equity for values of EIS smaller than one. Because the equity premium is decreasing with EIS in the presence of recovery, dividend leverage ($\alpha > 0$) helps match qualitatively and quantitatively the observed term-structures of equity and the average level of the equity premium.

There is a debate about the values of EIS used in theoretical models and those estimated using real data (see Hall (1988) and Epstein, Farhi, and Strzalecki (2014)). Indeed, theoretical models are usually very sensitive to the EIS and generate realistic price patterns only under specific restrictions on EIS (e.g., Bansal and Yaron (2004)). In this section we show that our model can simultaneously capture both the timing and the level of equity risk premia. In what follows, we want to verify whether our results are robust to changes in preference parameters, and in particular to changes in the EIS. Figure 11 shows the pairs of relative risk aversion and time-discount factor (γ, δ) , which allow to match the empirical levels of the risk-free rate and the equity risk premium for a broad range of EIS values. Specifically, the range for the EIS covers the values considered in the literature ($\psi \in (1/2, 2)$), and we require a risk-free rate of 0.6% and an equity premium of 6.5% (Constantinides and Ghosh, 2011).

A number of results are noteworthy. First, the model can quantitatively solve the risk-free rate and equity premium puzzle for any value of EIS within this range. Second, the pairs (γ, δ) required by the model belong to the ranges $(\delta \in (0.964, 0.971))$ and $(\gamma \in (4.1, 8.5))$, which are considered to be plausible in the existing literature. Because in the presence of

¹¹The model leads to relatively low equity volatility (about 9.5%) because we do not include idiosyncratic risk in dividend dynamics.

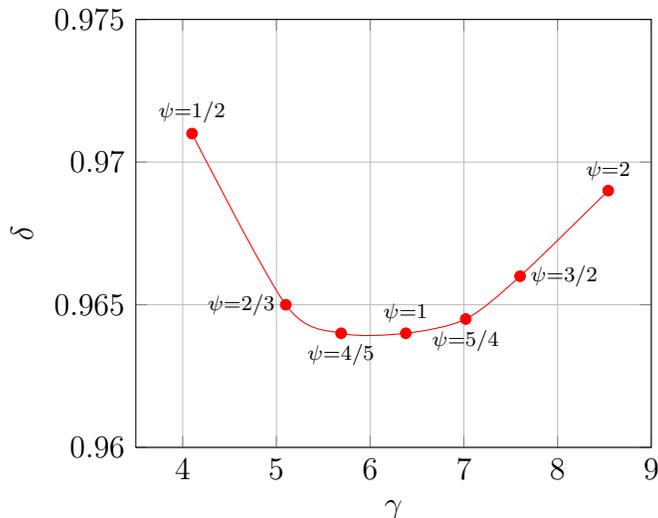


Figure 11: Risk-free rate and equity premium vs. preference parameters: robustness.

The figure displays the pairs (γ, δ) required by the model to match the risk-free rate $r = 0.6\%$ and the equity premium $RP = 6.5\%$ for each $\psi \in (1/2, 2)$. The calibration is provided in Table 2.

recovery the equity premium is decreasing with EIS, an increase in both EIS and relative risk aversion is needed to match the observed equity premium. Third, for each value of EIS within this range, the pair (γ, δ) required to match the observed risk-free rate and equity premium also implies downward-sloping term-structures of equity. That is, the speed of recovery from disasters implies a shift of equity risk towards the short-term that is robust to a wide range of preference parameters. This provides further support to the idea that recovery from disasters in macroeconomic fundamentals is a key driver of asset prices.

7 Conclusion

Empirical research provides evidence that the term-structures of equity volatility and risk premia are downward-sloping. This finding is particularly important provided that leading asset-pricing models (Bansal and Yaron, 2004; Gabaix, 2012; Wachter, 2013) imply strongly upward-sloping term-structures of equity. In this paper, we show that the term-structures of equity obtained in a rare disasters model are downward-sloping when accounting for post-disaster dividend recoveries—a feature of the data disregarded in the literature. In the presence of recoveries, disaster risk concentrates in the short-term, which implies downward-sloping term-structures of dividend risk, equity volatility, and equity risk premia.

When accounting for the observed upward-sloping risk of consumption (due to time-varying expected growth), the downward-sloping risk of dividends (due to recovery from disas-

ters), and the co-integrating relationship between dividends and consumption, the model reconciles a high equity premium and a low risk-free rate with downward-sloping term-structures of equity. We show that this result is obtained for a broad range of reasonable preference parameters, and in particular for both low and high values of the elasticity of intertemporal substitution.

A Appendix

A.1 Proof of Proposition 1

Following Eraker and Shaliastovich (2008), the state-price density satisfies

$$d \log M_t = (\theta \log \delta - (\theta - 1) \log k_1 + (\theta - 1)(k_1 - 1)B'(Y_t - \mu_Y)dt - \Omega' dY_t,$$

where $\mu_Y = (0, 0, \lambda_m)^\top$ and $\Omega = \gamma(1, 1, 0)^\top + (1 - \theta)k_1 B$.

The coefficients $A \in \mathbb{R}$, $B \in \mathbb{R}^3$, and $k_1 \in \mathbb{R}$ characterizing the stock price, P , and the state-price density, M , solve the following system of equations:

$$\begin{aligned} 0 &= \mathcal{K}^\top \chi - \theta(1 - k_1)B + \frac{1}{2}\chi^\top H\chi + l_1^\top (\varrho(\chi) - 1) \\ 0 &= \theta(\log \delta + k_0 - (1 - k_1)A) + \mathcal{M}^\top \chi + \frac{1}{2}\chi^\top h\chi + l_0^\top (\varrho(\chi) - 1) \\ \theta \log k_1 &= \theta(\log \delta + (1 - k_1)B^\top \mu_Y) + \mathcal{M}^\top \chi + \frac{1}{2}\chi^\top h\chi + l_0^\top (\varrho(\chi) - 1), \end{aligned}$$

where $\chi = \theta \left((1 - \frac{1}{\psi})(1, 1, 0)^\top + k_1 B \right)$, $k_0 = -\log \left((1 - k_1)^{1-k_1} k_1^{k_1} \right)$.

□

A.2 Proof of Proposition 2

Following Eraker and Shaliastovich (2008), the coefficient $\Phi_0 \in \mathbb{R}$ and $\Phi_1 \in \mathbb{R}^3$ characterizing the risk-free rate r_t solve the following system of equations:

$$\begin{aligned} \Phi_1 &= (1 - \theta)(k_1 - 1)B + \mathcal{K}^\top \Omega - \frac{1}{2}\Omega^\top H\Omega - l_1^\top (\varrho(-\Omega) - 1) \\ \Phi_0 &= -\theta \log \delta + (\theta - 1)(\log k_1 + (k_1 - 1)B^\top \mu_Y) + \mathcal{M}^\top \Omega - \frac{1}{2}\Omega^\top h\Omega - l_0^\top (\varrho(-\Omega) - 1). \end{aligned}$$

□

A.3 Proof of Proposition 3

Following Eraker and Shaliastovich (2008), the deterministic functions $a(\cdot) \in \mathbb{R}$ and $b(\cdot) \in \mathbb{R}^3$ characterizing the price of the dividend strip $S_t(\tau)$ solve the following system of ordinary differential equations:

$$\begin{aligned} b'(\tau) &= -\Phi_1 + \mathcal{K}^{\mathbb{Q}\top} b(\tau) + \frac{1}{2}b(\tau)^\top Hb(\tau) + l_1^{\mathbb{Q}\top} (\varrho^{\mathbb{Q}}(b(\tau)) - 1) \\ a'(\tau) &= -\Phi_0 + \mathcal{M}^{\mathbb{Q}\top} b(\tau) + \frac{1}{2}b(\tau)^\top hb(\tau) + l_0^{\mathbb{Q}\top} (\varrho^{\mathbb{Q}}(b(\tau)) - 1) \end{aligned}$$

subject to $a(0) = 0$ and $b(0) = (1, 1, 0)^\top$. The coefficients $\mathcal{M}^\mathbb{Q} \in \mathbb{R}^3$, $\mathcal{K}^\mathbb{Q} \in \mathbb{R}^{3 \times 3}$, $l_0^\mathbb{Q} \in \mathbb{R}^3$, $l_1^\mathbb{Q} \in \mathbb{R}^{3 \times 3}$, and $\varrho^\mathbb{Q}(\cdot) \in \mathbb{R}^3$ satisfy

$$\begin{aligned} \mathcal{M}^\mathbb{Q} &= \mathcal{M} - h\Omega & \mathcal{K}^\mathbb{Q} &= \mathcal{K} - H\Omega \\ l^\mathbb{Q}(Y_t) &= l(Y_t) \cdot \varrho(-\Omega) \equiv l_0^\mathbb{Q} + l_1^\mathbb{Q} Y_t & \varrho^\mathbb{Q}(u) &= \varrho(u - \Omega) / \varrho(-\Omega). \end{aligned}$$

□

A.4 Proof of Proposition 5

Following [Duffie, Pan, and Singleton \(2000\)](#), the deterministic functions $\bar{a}(\cdot; \cdot) \in \mathbb{R}$ and $\bar{b}(\cdot; \cdot) \in \mathbb{R}^3$ characterizing the moment-generation function $\text{MGF}(\cdot, \cdot; \cdot)$ solve the following system of ordinary differential equations:

$$\begin{aligned} \bar{b}'(\tau; u) &= \mathcal{K}^\top \bar{b}(\tau; u) + \frac{1}{2} \bar{b}(\tau; u)^\top H \bar{b}(\tau; u) + l_1^\top (\varrho(\bar{b}(\tau; u)) - 1) \\ \bar{a}'(\tau; u) &= \mathcal{M}^\top \bar{b}(\tau; u) + \frac{1}{2} \bar{b}(\tau; u)^\top h \bar{b}(\tau; u) + l_0^\top (\varrho(\bar{b}(\tau; u)) - 1), \end{aligned}$$

subject to $a(0) = 0$ and $b(0) = (u, u, 0)^\top$.

□

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