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# *Screening as a Unified Theory of Delinquency, Renegotiation, and Bankruptcy\**

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## **Abstract**

We propose a parsimonious model with adverse selection where delinquency, renegotiation, and bankruptcy all occur in equilibrium as a result of a simple screening mechanism. A borrower has private information about her cost of bankruptcy, and a lender may use random contracts to screen different types of borrowers. In equilibrium, some borrowers choose not to repay, and thus become delinquent. The lender renegotiates with some delinquent borrowers. In the absence of renegotiation, delinquency leads to bankruptcy. We apply the model to analyze effects of a government intervention in debt restructuring. We show that a mortgage modification program aimed at limiting foreclosures that fails to take into account private debt restructuring may have the opposite effect from the one intended.

**Keywords:** Default, Delinquency, Bankruptcy, Renegotiation, Adverse Selection, Screening, Consumer Credit

**JEL Codes:** D14, D82, D86, G18, G21

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# 1 Introduction

Default in consumer credit markets is not a simple binary event, but rather has multiple stages and possible outcomes. The first “stage” is delinquency, which is defined as being overdue on loan payments for a specified period of time (usually at least 60 days). Some, but not all, delinquent borrowers end up in bankruptcy. Lenders sometimes renegotiate with delinquent borrowers to prevent bankruptcy and achieve debt settlement.

We propose a very simple model where a single key friction generates all three phenomena — delinquency, renegotiation, and bankruptcy — as parts of an optimal arrangement. The friction is adverse selection — a borrower has private information about her cost of bankruptcy. We assume that the borrower is indebted to a single lender. To keep the model as simple as possible, we abstract from how the debt was acquired.<sup>1</sup> The lender offers repayment options to the borrower and seeks to maximize the expected repayment. The lender cannot ask for a repayment that is higher than the debt level. The alternative for the borrower to making the repayment is to file for bankruptcy. We focus on the case where the borrower’s cost of bankruptcy, unknown to the lender, can take one of two values, high or low.

We first consider a situation where debt is so high that it does not restrict the lender’s optimal choice of repayments. We will refer to this scenario as “debt overhang”. Faced with adverse selection, the lender has two basic options when restricted to offering deterministic contracts. First, by asking for repayment that does not exceed the low-cost borrower’s willingness to pay, the lender can guarantee repayment from both types. Second, by asking for a greater repayment, the lender can extract more from the high-cost borrowers, but loses the low-cost type to bankruptcy.

The lender may be able to do better if he extracts different repayments from different types of borrowers. However, he cannot separate the two types of borrowers by offering a menu of deterministic contracts. The reason is that both types of borrowers have the same utility if they make the same repayment, so naturally every borrower will choose a lower repayment. But different types do have different utilities if they do *not* repay and end up in bankruptcy. The lender can utilize this feature and separate the two types of borrowers by using lotteries over repayments and bankruptcy. That is, the separation is possible because the two types of borrowers value lotteries in a different way, as their cost

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<sup>1</sup>Endogenously determined debt can be easily incorporated into the model, as we show in the Appendix. Focusing on a single-period setup with exogenous debt highlights the simplicity of our mechanism and allows us to illustrate our results in the most parsimonious model possible.

of bankruptcy is different.

We show that the optimal screening mechanism involves the lender offering a menu of random contracts that consists of a deterministic repayment and a lottery, aimed at the high- and low-cost borrowers, respectively. The lottery for the low-cost type is over a repayment that is lower than the deterministic one, and a very high repayment that exceeds the willingness to pay of both types. In this optimal mechanism, the high-cost borrowers make a higher repayment, while the low-cost borrowers decline that repayment and are then offered a better deal with some probability, but are forced into bankruptcy with the complementary probability.

Next, we consider a scenario when debt does restrict the lender's offers, i.e., there is no debt overhang. We show that when the debt level is below the low-cost borrowers' willingness to pay, all borrowers repay their debt in full. However, when debt exceeds this threshold (but there is still no debt overhang) and the fraction of high-cost borrowers is high enough, the optimal mechanism involves screening via random contracts.

One of the central points of the paper is that such a mechanism has a natural economic interpretation and delivers the three phenomena — delinquency, renegotiation, and bankruptcy — described above. Indeed, offering the aforementioned menu is equivalent to making the following sequential offers. First, the lender offers a high repayment that only the high-cost borrowers accept. As long as there is no debt overhang, this high repayment exactly equals the face value of debt. We interpret the borrowers who have agreed to make the high payment as having repaid the loan, while the borrowers who refuse to make it as becoming *delinquent*. Next, the lender offers a lower repayment to a fraction of the delinquent borrowers. We interpret the event of offering the lower repayment as *renegotiation*. The delinquent borrowers with whom the lender does not renegotiate declare *bankruptcy*.<sup>2</sup>

Renegotiation allows the lender to extract some repayment from the low-cost borrowers who reject the high repayment. However, the possibility of renegotiation makes delinquency more attractive and thus limits the amount that can be extracted from the high-cost borrowers. It is for this reason that the lender does not renegotiate with all delinquent borrowers. Thus, our paper also addresses the question of why we see some renegotiation in the consumer credit market but not all bankruptcies are avoided.

We illustrate that our model generates reasonable comparative statics predictions. In

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<sup>2</sup>An implicit assumption needed for the sequential offers to be equivalent to the (simultaneous) menu offer is that the lender can commit not to renegotiate with all delinquent borrowers, for otherwise the high-cost borrowers will never agree to make the initial high repayment.

particular, we show that the bankruptcy rate is increasing in the debt level and is decreasing in the borrower’s income.

Our model puts us in the unique position to analyze effects of a government intervention in consumer debt restructuring. One example of such an intervention is a mortgage modification program aimed at limiting foreclosures. Indeed, for an individual borrower, such an intervention is triggered by *delinquency*, offers debt restructuring — i.e., involves a *renegotiation*, — with the goal of avoiding *bankruptcy*, or foreclosure. Not only does our model capture all these stages of default, but, most importantly, it allows us to explicitly analyze the response of private lenders to the government intervention. We show that a government program that fails to take into account private debt restructuring may have the opposite effect from the one intended — rather than limiting the number of foreclosures, it may actually increase it. We also demonstrate how a seemingly irrelevant intervention can successfully prevent *all* defaults. Our analysis therefore illustrates that it is crucial for a policy maker designing such a program to take into account how private debt restructuring works, or else the program may backfire.

The rest of the paper is organized as follows. The next subsection reviews the related literature. Section 2 sets up the model. Section 3 characterizes the optimal contract and discusses how the screening mechanism captures the three stages of default. Section 4 presents the comparative statics results. Section 5 analyzes the effects of a government intervention. Section 6 concludes.

## 1.1 Related Literature

Theoretical analysis of default in consumer credit markets has largely focused on bankruptcy and abstracted from delinquency, and especially renegotiation — see, for example, Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007), Livshits, MacGee, and Tertilt (2007), and many others. Notable exceptions are the papers by Chatterjee (2010), Adelino, Gerardi, and Willen (forthcoming), Benjamin and Mateos-Planas (2012) and Athreya, Sanchez, Tam, and Young (2012). While Chatterjee (2010) makes a distinction between delinquency and bankruptcy, he does not allow for renegotiation.<sup>3</sup> Adelino, Gerardi, and Willen (forthcoming), on the other hand, study renegotiation, but treat delinquency as exogenous. They

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<sup>3</sup>The distinction between bankruptcy and “informal bankruptcy” is also present in Dawsey and Ausubel (2004) and Dawsey, Hynes, and Ausubel (2009), but the informal bankruptcy is thought of as a terminal state, much like bankruptcy, rather than as a transitional stage that delinquency captures.

document that renegotiations of delinquent mortgages are infrequent.<sup>4</sup> In explaining this phenomenon, the authors point out that mortgage restructuring may not be ex-post profitable for the lenders as it foregoes to possibility of “self-cures” — delinquent mortgages being repaid in full. In contrast, in our model renegotiation is always profitable ex-post (i.e., after the borrower becomes delinquent, but generates an ex-ante distortion by affecting the incentive of the high-cost borrowers to make the high repayment rather than choose delinquency. Thus, we view our explanation for why lenders do not renegotiate more frequently as complementary to that offered by Adelino, Gerardi, and Willen (forthcoming).

Benjamin and Mateos-Planas (2012) and Athreya, Sanchez, Tam, and Young (2012) propose quantitative models with symmetric information and incomplete markets where all three stages of default are present. However, the mechanics of their models are very different from ours. In Benjamin and Mateos-Planas (2012), renegotiation occurs with an exogenously given probability, but the possibility of renegotiation leads to an endogenous distinction between delinquency and bankruptcy. In Athreya, Sanchez, Tam, and Young (2012), delinquency also triggers debt restructuring, but deterministically so.<sup>5</sup> In contrast, in our model, the probability of renegotiation following delinquency is determined endogenously, and, as will be clear from Section 5, the endogeneity of renegotiation is crucial for policy analysis.

Another related paper is Hopenhayn and Werning (2008), who study a dynamic lending model where, like in our paper, the borrower has private information about her outside option. The optimal contract in their framework also features default occurring in equilibrium with positive probability. However, their model does not distinguish between delinquency and default (which is akin to bankruptcy in our setup), and thus does not allow for the possibility of renegotiation.

The reason for delinquency in our model is distinct from the consumption-smoothing motive in Herkenhoff (2012), where delinquency (“informal default”) is basically a temporary reprieve for borrowers with negative income shocks while they wait for their incomes to recover, at which point they make a full repayment of the rolled-over debt.<sup>6</sup> Since in our model there is only one period (and thus no future income shocks), delinquency never results in a full repayment. While “self-curing” delinquencies are clearly present in the data,<sup>7</sup>

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<sup>4</sup>Agarwal, Amromin, Ben-David, Chomsisengphet, and Evanoff (2011) also point out that lenders restructure merely a small fraction of delinquent mortgages.

<sup>5</sup>Coexistence of bankruptcy and delinquency in Athreya, Sanchez, Tam, and Young (2012) arises from an exogenously imposed additional cost of delinquency, namely income garnishment.

<sup>6</sup>A similar mechanism is also present in Chatterjee (2010).

<sup>7</sup>Herkenhoff and Ohanian (2012) offer a nice summary of empirical facts regarding transitions of mort-

this paper does not attempt to explain them and focuses on a complementary mechanism instead.

Unlike in the consumer debt literature, analysis of renegotiation has played a central role in the sovereign debt literature — see the seminal work by Bulow and Rogoff (1989) and more recent contributions by Kovrijnykh and Szentes (2007), Benjamin and Wright (2009), Yue (2010), Arellano and Bai (2012), and others. Our work differs from this strand of literature in a number of ways. One distinction is that the key friction in our paper is private information about the bankruptcy cost, which arguably is more relevant in consumer debt than sovereign debt context. Also, unlike the sovereign default papers, our model allows us to study an “extensive margin” of renegotiation, as the fraction of borrowers with whom the lender renegotiates is determined endogenously. This in turn allows us to analyze the effect of an intervention operating along this extensive margin.

From the modeling standpoint, our paper is closely related to papers by Maskin and Riley (1984), Matthews (1983), and Miller, Piankov, and Zeckhauser (2005). Maskin and Riley (1984) study a problem of designing an auction that maximizes the expected revenue of a seller of an indivisible good facing risk-averse bidders with unknown preferences. They show that making buyers bear risk relaxes incentive constraints. In addition, they find that the probability of winning the auction (obtaining the good) and the amount paid in the case of winning increase with a buyer’s valuation. Our result is similar in that, in our screening contract, a low-cost borrower makes a lower repayment, and with a lower probability, than a high-cost borrower. Matthews (1983) studies a similar problem to the one analyzed by Maskin and Riley (1984), but also analyzes the case where there is an unlimited supply of indivisible units sold. This case is closer to our setup, where it is possible for the lender to obtain repayments (which is analogous to selling a good) from multiple borrowers. Matthews (1983) finds that the optimal selling scheme gives some buyers only a probability of obtaining the good. Finally, Miller, Piankov, and Zeckhauser (2005) also consider a similar setup as the other two papers, but have the seller making sequential price offers. They show that the optimal selling scheme involves the seller making an offer that, if rejected, is followed by a subsequent, more attractive offer, but only with some probability. This selling scheme is similar to the sequential interpretation of the optimal contract in our model.

There are, however, important differences between our setup and the ones considered in these papers. First, in our model, different types of borrowers have identical payoffs 

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gages into and out of delinquencies.

from repaying but different payoffs from not repaying (declaring bankruptcy). In contrast, in the papers described above, the buyers differ in their utilities from obtaining the good, but derive identical utilities from not getting it. Thus, it may be possible to screen the buyers using lotteries over payments while selling a unit to each buyer with probability one (for instance, if there is unlimited supply of units, as in Matthews, 1983). In contrast, in our setup screening must involve some borrowers exercising their outside option. Second, these papers impose interim participation constraints, while in our paper the borrower can refuse to “participate” in the mechanism *ex post*, i.e., after the outcome of a lottery is realized. Notably, our application of screening through randomization to the environment of consumer credit generates a novel, unified theory of delinquency, renegotiation, and bankruptcy.

Finally, our analysis of the government intervention in debt restructuring contributes to the literature on the effects of the most notable such intervention in recent years — the Home Affordable Mortgage Program (HAMP), aimed at restructuring troubled mortgages and preventing foreclosures, which has been in place in the U.S. since 2009. Agarwal, Amromin, Ben-David, Chomsisengphet, Piskorski, and Seru (2012) offer a comprehensive empirical analysis of the effects of this program. The authors highlight the importance of accounting for changes in private restructuring in evaluating the effects of the program. Our theoretical model allows us to explicitly analyze the private sector’s response to an intervention, and to illustrate that it can lead to unexpected, and possibly undesired, consequences. These insights are complementary to the existing studies pointing out possible shortcomings of HAMP. Most notably, Mulligan (2009, 2010) points out severe distortions imposed by the means-testing aspect of the program that induces an excessively high effective income tax rate. Specifically, since the restructured payments depend directly on the borrower’s income, HAMP creates a strong incentive for the borrower to earn less. We treat income of borrowers as exogenous, thus ignoring such distortions. Instead, we highlight the distortions imposed by such a government program on the private sector debt renegotiation.

## 2 The Environment

We begin by studying a simple one-period environment with one lender and one borrower.<sup>8</sup> The borrower is risk averse, and derives utility from consumption according to the utility

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<sup>8</sup>We can alternatively assume that there are many borrowers.

function  $u : (0, +\infty) \rightarrow \mathbb{R}$ . The function  $u$  is continuous, strictly increasing, strictly concave, and satisfies the Inada condition  $\lim_{c \rightarrow 0} u'(c) = +\infty$ . The borrower has endowment  $I$ , known to everyone. We denote the level of debt that the borrower owes to the lender by  $D$ . We abstract from where the debt comes from. While endogenous debt can be easily incorporated into the model as illustrated in the Appendix, focusing on the single-period setup with exogenous debt highlights the simplicity of the mechanism we propose.

As an alternative to making repayments to the lender, the borrower has an option of declaring bankruptcy. The borrower's cost of bankruptcy can be low or high,  $\theta \in \{\theta_L, \theta_H\}$ , where  $\theta_L < \theta_H$ . This cost is known to the borrower, but is unobservable to the lender.<sup>9</sup> The prior belief of the lender that the bankruptcy cost is high is denoted by  $\gamma$ , where  $0 \leq \gamma \leq 1$ . Alternatively,  $\gamma$  can be interpreted as the fraction of high-cost borrowers. If the borrower declares bankruptcy, she receives utility  $v(I, \theta)$ , while the lender receives nothing. The function  $v(I, \theta)$  is strictly increasing in  $I$  for each  $\theta$ , and  $v(I, \theta_H) < v(I, \theta_L)$ . Moreover,  $u(0) < v(I, \theta) < u(I)$  for all  $I$  and  $\theta$ .

The lender is risk neutral and maximizes the expected repayment that he extracts from the borrower. We assume that the lender makes a take-it-or-leave-it offer to the borrower. An offer consists of a menu of contracts, where each contract — which can be deterministic or random — specifies how much the borrower should repay to the lender. A deterministic contract is simply an amount  $R$  that the borrower is asked to repay; a random contract is a lottery over repayments. We assume that the lender cannot ask the borrower to repay more than the amount of debt owed; that is, the repayment cannot exceed  $D$ . The borrower chooses one contract from the offered menu or rejects all contracts. In the latter case (or if the borrower does not make the repayment specified in the contract he chose) she has to declare bankruptcy.

### 3 Optimal Contracts

In this section we will describe the optimal solution to the lender's problem. Before considering possible contracts that the lender can offer in equilibrium, it will be useful to define

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<sup>9</sup>There are many examples of differences in bankruptcy costs across borrowers that are unobservable to lenders. For example, borrowers planning to move to a new city and find a new job will tend to have a higher cost of bankruptcy, as their potential new employers and landlords are likely to check their credit reports. On the other hand, borrowers who can rely on their family and friends for support in the event of bankruptcy (e.g., ones willing to move in with their parents) will have a lower cost of bankruptcy. In application to mortgages, borrowers vary in how they value their houses (how well a house matches a family's current need, how much the borrower invested in customizing the house, etc.).

$R_j(I)$  — the largest amount that a borrower with income  $I$  and bankruptcy cost  $\theta_j$  is willing to repay. This repayment solves

$$u(I - R_j(I)) = v(I, \theta_j), \tag{1}$$

$j \in \{L, H\}$ . By construction, the “willingness to repay” of the low-cost borrowers is lower than that of the high-cost borrowers:  $R_L \leq R_H$ .

It will be convenient to start by analyzing the case of debt overhang case, defined as the situation where the debt is so large that it does not restrict the contracts that the lender may offer. After characterizing the optimal contracts in this simpler case, we will return to the original problem where repayments may be constrained by the amount of debt.

## 3.1 The Debt Overhang Case

### 3.1.1 Deterministic Contracts

Suppose first that the lender is restricted to offering a single deterministic contract. Depending on the level of the demanded repayment, denoted by  $R$ , three situations may arise. If  $R \leq R_L$ , then both types of borrowers will accept the contract. If  $R \in (R_L, R_H]$ , then only the high-cost borrowers will accept the contract, while low-cost borrowers will prefer to declare bankruptcy. Finally, if  $R > R_H$ , no borrower will accept the contract. Therefore, to maximize the expected repayment, the lender will offer either  $R = R_L$  or  $R = R_H$ . We will refer to the first alternative as “pooling”, as it attracts both types of borrowers, and to the second one as “exclusion”, as it excludes — i.e., forces into bankruptcy — the low-cost borrowers.

Which of the two contracts generates higher profits to the lender will depend on the parameters of the model, in particular, on the fraction of the high-cost borrowers,  $\gamma$ , and the extent to which the bankruptcy cost parameters,  $\theta_H$  and  $\theta_L$ , are different from each other. Specifically, the lender prefers exclusion to pooling whenever  $\gamma > R_L/R_H$ , where the values on the right-hand side are completely pinned down by the primitives of the model (see equation (1)).

### 3.1.2 Random Contracts

Since a deterministic contract specifies only the repayment, it is impossible to offer a menu of deterministic contracts and have different types of borrowers accepting different

contracts. However, the lender may be able to achieve this by offering a menu of random contracts, as we will demonstrate below. We will refer to this case as “screening”, as the lender uses lotteries to screen the borrowers based on their cost of bankruptcy. As we only have two types of borrowers, we can, without loss of generality, limit the analysis to just two random contracts.

It is straightforward to see that the expected repayment is maximized by offering the following pair of contracts. The first contract is deterministic with repayment, which we denote by  $R_S$ , that attracts only the high-cost borrowers. The second contract is a lottery that offers a lower repayment with probability  $p$  and an implausibly large repayment (anything above  $R_H$ ) with probability  $1 - p$ .<sup>10</sup> To maximize the lender’s expected profit, the lower repayment in the second contract must be set to  $R_L$ : it maximizes the repayment extracted from the low-cost borrowers, and also minimizes the attractiveness of this contract to the high-cost borrowers. We denote the lottery by  $(R_L, p)$ .

Note that the only reason for  $p$  to be set strictly below one is to keep the high-cost borrowers from accepting the contract meant for the low-cost borrowers: if  $p$  were equal to one, the high-cost borrowers would never make the higher repayment offered to them. Indeed, profit maximization requires the deterministic repayment  $R_S$  to be such that the high-cost type is just indifferent between the two contracts. That is,

$$u(I - R_S) = pu(I - R_L) + (1 - p)v(I, \theta_H) = pu(I - R_L) + (1 - p)u(I - R_H), \quad (2)$$

where the second equality follows from (1). Clearly,  $R_S$  is lower than  $R_H$  as long as  $p > 0$ , as offering the lottery will prevent extracting the full surplus from the high-cost type. Also,  $R_S$  is higher than  $R_L$  as long as  $p < 1$ , for otherwise the high-cost borrower’s incentive constraint is lax and the lender could increase expected repayment by increasing  $R_S$ .

The lender’s problem is then simply to choose  $p$  to maximize the expected repayment,

$$\max_{p \in [0,1]} \gamma R_S(p) + (1 - \gamma)pR_L, \quad (3)$$

where  $R_S(p)$  is given by (2). Notice that choosing  $p = 1$  and  $p = 0$  corresponds to the pooling and exclusion scenarios, respectively. Therefore, the lender’s problem is fully captured by the maximization problem (3) subject to constraint (2). Strict concavity of the utility function immediately implies that this problem has a unique solution, which we

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<sup>10</sup>Offering such an implausibly large repayment is equivalent to simply offering the borrower the bankruptcy option.

denote by  $p^*$ . The corresponding repayment by the high-cost type,  $R_S(p^*)$ , is denoted by  $R_S^*$ . We summarize the above discussion in the following proposition.

**Proposition 1** *The repayment scheme that maximizes the lender's profits is to offer a menu consisting of a deterministic repayment  $R_S^*$  and a lottery  $(R_L, p^*)$ , where  $p^*$  solves (3) subject to (2).*

### 3.1.3 Screening and Risk Aversion

We have described three possible strategies that the lender may follow: pooling, exclusion, and screening. Given the focus of the paper, the screening scenario is the most interesting of the three. Then the question arises: does the lender ever use screening — i.e., chooses  $p \in (0, 1)$  — in equilibrium?

Interestingly, if borrowers were risk neutral, lotteries (and hence screening) would never be utilized in equilibrium. To see this, notice that with a linear utility function, equation (2) reduces to  $R_S = pR_L + (1 - p)R_H$ , and the lender's problem becomes

$$\max_{p \in [0, 1]} pR_L + (1 - p)\gamma R_H.$$

Notice that the profits in the objective function is simply a linear combination of the profits under pooling and exclusion. That is, screening is always dominated by either pooling or exclusion (strictly so, unless  $R_L = \gamma R_H$ ). Thus, the lender does not benefit from using random contracts.

With risk-averse borrowers, however, there are parameter values for which screening gives the lender a strictly higher payoff than the pooling and exclusion alternatives. This happens, for example, when  $R_L = \gamma R_H$ . At that point, the lender is indifferent between pooling and exclusion, as well as any screening menu consisting of the lottery  $(p, R_L)$  and the deterministic offer  $\bar{R}(p) = pR_L + (1 - p)R_H$ . Note that the low-cost borrowers are not affected by the riskiness of the lottery, as both outcomes generate the same utility for them (equal to their value of bankruptcy). Note further that a risk-neutral high-cost borrower would have been indifferent between the lottery  $(p, R_L)$  and the deterministic offer  $\bar{R}(p)$ . A risk-averse high-cost borrower, however, strictly prefers the latter, and thus the lender is able to extract a higher payment  $R_S(p) > \bar{R}(p)$  from her. As a result, the expected repayment is maximized by choosing some interior  $p \in (0, 1)$ .

Of course, there are parameter values for which either pooling or exclusion would be

the lender's optimal strategies. In particular, exclusion (pooling) is attractive when  $\gamma$  is high (low) enough.

### 3.2 The General Case

So far, we have characterized the optimal contracts in the case of debt overhang, when debt is so high that its level does not constrain the lender and is thus irrelevant for equilibrium repayments. Note that the debt overhang occurs when  $D \geq R_S^*$ , as  $R_S^*$  is the highest repayment demanded by the unconstrained lender. We now turn to the general case with an arbitrary level of debt.

The lender's problem in the general case is easily obtained from the unconstrained problem (2)–(3) by simply imposing additional constraints that the offered repayments cannot exceed  $D$ . Specifically, let  $R_L^D = \min\{R_L, D\}$ . Then the lender's problem becomes

$$\max_{p \in [0,1], R_S^D} \gamma R_S^D + (1 - \gamma)pR_L^D, \quad (4)$$

$$\text{s.t.} \quad u(I - R_S^D) = pu(I - R_L^D) + (1 - p)u(I - R_H), \quad (5)$$

$$R_S^D \leq D. \quad (6)$$

Let  $p_D^*$  denote the lender's optimal choice of  $p$  in this generalized problem. Note that when constraint (6) binds,  $p_D^*$  is pinned down by equation (5):

$$p_D^* = \frac{u(I - D) - u(I - R_H)}{u(I - R_L^D) - u(I - R_H)}. \quad (7)$$

When  $D \leq R_L$ , the above problem simply delivers  $R_S^D = D$  and  $p_D^* = 1$ , which is a pooling contract where all borrowers fully repay their debt. When  $D \geq R_S^*$ , (6) does not bind and thus the solution is the same as in the debt overhang case,  $R_S^D = R_S^*$  and  $p_D^* = p^*$ . The more interesting case is when  $D \in (R_L, R_S^*)$ .<sup>11</sup> In this case  $R_L^D = R_L$ , and since (6) binds,  $R_S^D = D$ . Furthermore, as  $R_L < D < R_S^* \leq R_H$ , equation (7) implies  $p_D^* \in (0, 1)$ . That is, the lender performs screening where the high-cost borrowers fully repay their debt, while the fraction  $p_D^*$  of delinquent borrower receive an offer with a lower repayment of  $R_L$ . In particular,  $p_D^* > 0$  means that the constrained lender never performs exclusion. Intuitively, suppose the lender chooses to perform exclusion under debt overhang (i.e., he extracts  $R_H$  from the high-cost borrowers). He does not find screening attractive because

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<sup>11</sup>Note that this interval is empty if the lender chooses pooling under debt overhang, i.e., when  $R_S^* = R_L$ .

the expected repayment from the low-cost borrowers is not enough to offset the decrease in the repayment from the high-cost borrowers. But when the debt level is between  $R_L$  and  $R_H$ , the lender can only extract  $D$  from them anyway (i.e.,  $R_S^D = D$ ). Therefore, he might as well offer  $R_L$  to the low-cost type, and pick the probability of renegotiation that makes the high-cost type just indifferent between the two offers.<sup>12</sup>

Note also that since  $p^*$  solves (7) when  $D = R_S^*$ , the probability  $p_D^*$  for  $D \in (R_L, R_S^*)$  always exceeds  $p^*$ . In other words, a constrained lender sends a smaller fraction of borrowers to bankruptcy than an unconstrained lender. Moreover, for  $D \in (R_L, R_S^*)$  equation (7) immediately implies that  $p_D^*$  is strictly decreasing in  $D$ , so the higher the debt, the lower the bankruptcy rate. (We will revisit this last result in Section 4.)

We summarize these findings in the proposition below. Figure 1 further illustrates the results by depicting the types of contracts offered by the lender depending on the level of debt and on what he would have offered in the debt overhang case. The figure also plots the probability of bankruptcy  $(1 - \gamma)(1 - p_D^*)$  as a function of the debt level  $D$ . In what follows, we will often refer to the probability of bankruptcy as the bankruptcy rate.

**Proposition 2** *(i) If  $D \geq R_S^*$ , there is debt overhang, and the lender offers  $(R_S^*, (R_L, p^*))$  that solves the unconstrained problem.*

*(ii) If  $D \leq R_L$ , the lender demands repayment  $D$ , and all borrowers fully repay their debt.*

*(iii) If  $D \in (R_L, R_S^*)$ , the lender performs screening. He offers  $R_S^D = D$  to the high-cost borrowers and  $R_L$  with probability  $p_D^* > p^*$  to the low-cost borrowers.*

To recap, when the face value of debt restricts how much the lender can extract from the borrower, the lender will never choose to go after the high-cost borrowers only and will necessarily extract some repayment from the low-cost borrowers. Furthermore, we obtain simple sufficient conditions under which screening is part of the optimal contract. Specifically, this happens whenever an unconstrained lender would not choose pooling (a sufficient condition for which is  $\gamma > R_L/R_H$ ) and the debt level is in the intermediate range  $(R_L, R_H)$ .<sup>13</sup> (Recall that  $R_L$  and  $R_H$  depend on the primitives of the model only.)

<sup>12</sup>Equation (2) provides a simple way to see this: if  $R_S \in (R_L, R_H)$ , then  $p \in (0, 1)$ .

<sup>13</sup>If the lender chooses screening under debt overhang, he will also use screening for all  $D > R_L$ . If the lender chooses exclusion under debt overhang, he will use screening for  $D \in (R_L, R_H)$ .

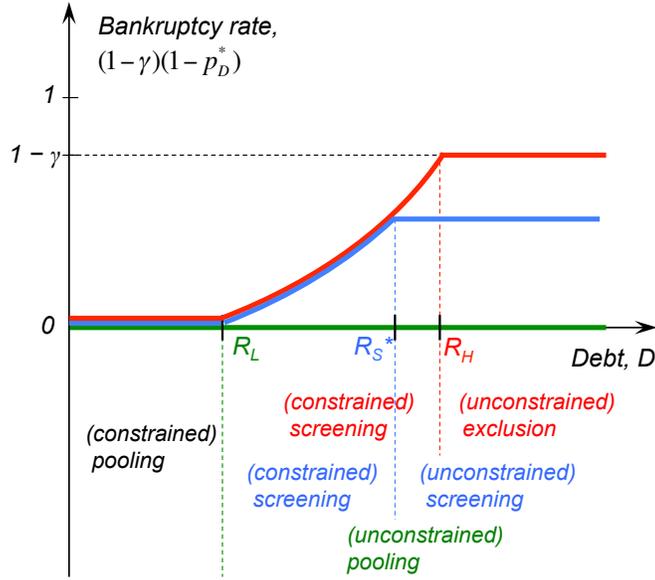


Figure 1: The probability of bankruptcy as a function of the debt level, and the corresponding types of equilibrium contracts. The three lines correspond to different parameter values generating three possible cases obtained under debt overhang: exclusion (red line), screening (blue), and pooling (green).

### 3.3 Sequential Interpretation of the Optimal Contract

One of the central points of the paper is that the simple screening mechanism described above generates the three stages of default in consumer credit — delinquency, renegotiation, and bankruptcy. In this subsection, we use a sequential setting to illustrate this point.

Suppose that instead of offering the two contracts simultaneously, the lender offers them sequentially. Assume also that the lender can commit ahead of time to (not) making offers. To be exact, he can commit to the probability of not making the second offer before the first offer is made. It is easy to see that under this assumption, the setup with sequential offers is equivalent to our original setup with simultaneous offers, and that the lender’s problem is still (4) subject to (5) and (6).

Consider the case when  $D \in (R_L, R_S^*)$  and suppose the lender chooses screening. In the sequential setting, the optimal screening contract has the following interpretation. First, the lender asks the borrowers to repay the debt in full (recall from part (iii) of Proposition 2 that  $R_S^D = D$  in this case), which only the high-cost borrowers agree to. We interpret the low-cost borrowers who refuse to repay the debt in full as *delinquent*. Next, the lender offers a lower repayment to — i.e., *renegotiates* with — delinquent borrowers, but only with

some probability. The borrowers with whom the lender renegotiates reach debt settlement, while the rest declare *bankruptcy*.<sup>14</sup>

Notice that the assumption of commitment is crucial here. Without it, the lender would want to renegotiate with all borrowers who refused to make the initial high repayment. Of course, anticipating this, no one would make the high repayment to begin with.

When the face value of debt is small enough ( $D \leq R_L$ ) all borrowers fully repay their debt, and delinquency and bankruptcy are altogether avoided. On the other hand, when the debt level is excessively large (so that there is debt overhang,  $D > R_S^*$ ), there is initial debt forgiveness for all borrowers, as the lender never asks the borrowers to repay  $D$ , only  $R_S^*$ .<sup>15</sup>

## 4 Comparative Statics

In this section, we establish some key comparative statics results. Specifically, we focus on how the equilibrium bankruptcy rate varies with the borrower's income and debt.

### 4.1 Comparative Statics with Respect to Debt

As we have discussed in Section 3.2, the bankruptcy rate is (weakly) increasing in the amount of debt. Therefore, the model generates reasonable comparative statics with respect to the debt level.

Specifically, Proposition 2 and Figure 1 illustrate that there are three “regions” of debt levels for a given income level. When debt is sufficiently low ( $D \leq R_L$ ), it is always repaid in full, and there is no bankruptcy (or delinquency) in equilibrium. When the face value of debt is sufficiently high ( $D \geq R_S^*$ ), the level of debt irrelevant, and thus does not affect the bankruptcy rate within this region. For intermediate levels of debt, the bankruptcy rate is strictly increasing in debt. We summarize these findings in the following claim.

**Claim 1** *The equilibrium bankruptcy rate is increasing in the level of debt, strictly increasing for  $D \in (R_L, R_S^*)$ .*

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<sup>14</sup>If there are more than two types of borrowers, settings with simultaneous and sequential offers are no longer equivalent. Nevertheless, the generalization to more than two types is straightforward. In the sequential setting, the lender screens different types by making offers with progressively lower repayments that are advanced to delinquent borrowers with a progressively lower probability. Thus, a borrower with a lower bankruptcy cost will have a longer expected delinquency duration than a higher-cost borrower.

<sup>15</sup>In this case, we will call delinquent borrowers refusing the repayment of  $R_S^*$ .

## 4.2 Comparative Statics with Respect to Income

We now turn to the analysis of how the equilibrium contracts as well as the bankruptcy rate change with the level of the borrower's income. In order to derive analytical results, we turn to specific functional forms of the utility function and the value of bankruptcy. In particular, we restrict our attention to the CRRA utility function,  $u(c) = c^{1-\sigma}/(1-\sigma)$ , and the bankruptcy cost being a fraction of income,  $v(I, \theta) = u((1-\theta)I)$ .

We begin by establishing the following intermediate result:

**Lemma 1** *Suppose that  $u(c) = c^{1-\sigma}/(1-\sigma)$  and  $v(I, \theta) = u((1-\theta)I)$ . Then in the case of debt overhang*

- (i) *Repayments  $R_L$ ,  $R_H$ , and  $R_S^*$  are proportional to the borrower's income;*
- (ii) *The probability of bankruptcy,  $(1-\gamma)(1-p^*)$ , is independent of the borrower's income.*

**Proof:** With  $v(I, \theta) = u((1-\theta)I)$ , equation (1) becomes  $u(I - R_j) = u((1-\theta_j)I)$ , which immediately implies that  $R_j = \theta_j I$  for  $j \in \{L, H\}$ . Furthermore, substituting  $u(c) = c^{1-\sigma}/(1-\sigma)$  into equation (2) and rearranging terms yields

$$\frac{(1 - R_S/I)^{1-\sigma}}{1-\sigma} = p \frac{(1 - \theta_L)^{1-\sigma}}{1-\sigma} + (1-p) \frac{(1 - \theta_H)^{1-\sigma}}{1-\sigma}.$$

Since the right-hand side of the above equation does not vary with  $I$ , the left-hand side does not either. Thus  $R_S$  is proportional to  $I$  for any  $p$ . Hence we can simply factor  $I$  out of the objective function (3), which implies that  $p^*$  does not depend on  $I$ . It then also follows that  $R_S^*$  is proportional to  $I$ .  $\square$

Lemma 1 shows that in the debt overhang case, if the lender faces a borrower with higher income he simply scales up the repayment(s) proportionally, but does not change the probability of renegotiation. Thus the bankruptcy rate is invariant to the borrower's income. This last result might sound undesirable at first glance as in reality high-income borrowers are presumably less likely to declare bankruptcy. Notice, however, that this result is established under debt overhang, i.e., for borrowers, whose debt exceeds  $R_S^*$ . And as part (i) indicates, this threshold is strictly increasing in the borrower's income. Since the bankruptcy rate is lower in the absence of debt overhang, a sufficiently large increase in income (keeping the debt level fixed) lowers the bankruptcy rate. Moreover, the bankruptcy rate is decreasing in income when debt is in the  $(R_L, R_S^*)$  region, as Claim 2 below establishes.

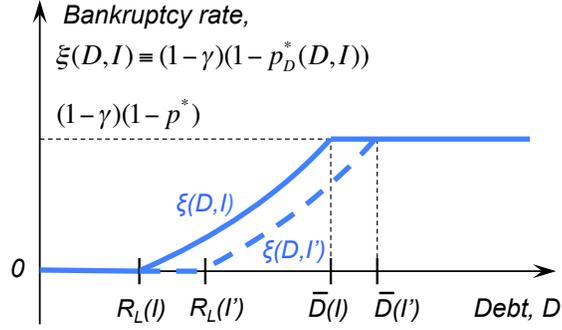


Figure 2: The probability of bankruptcy for two levels of income,  $I$  and  $I'$ , where  $I' > I$ .

**Claim 2** Suppose that  $u(c) = c^{1-\sigma}/(1-\sigma)$  and  $v(I, \theta) = u((1-\theta)I)$ . Then the probability of bankruptcy,  $(1-\gamma)(1-p_D^*)$ , is decreasing in the borrower's income  $I$  for any debt level  $D$ .

**Proof:** If  $D \geq R_S^*$  or  $D \leq R_L$ , then the statement holds trivially as the bankruptcy rate does not change with income in those regions — see Lemma 1. We want to establish that the bankruptcy rate decreases with income in the region of “constrained screening”, i.e., when  $D \in (R_L, R_S^*)$ . Since in that region constraint (6) is binding, constraint (7) (with  $R_L^D = R_L$ ) becomes

$$p_D^* = \frac{u(I-D) - v(I, \theta_H)}{v(I, \theta_L) - v(I, \theta_H)}.$$

(Note that  $p_D^*$  is strictly decreasing in  $D$ , just as Proposition 1 suggests.) For  $u(c) = c^{1-\sigma}/(1-\sigma)$  and  $v(I, \theta) = u((1-\theta)I)$ , the above equation becomes

$$p_D^* = \begin{cases} [(1-D/I)^{1-\sigma} - (1-\theta_H)^{1-\sigma}] / [(1-\theta_L)^{1-\sigma} - (1-\theta_H)^{1-\sigma}], & \text{if } \sigma \neq 1, \\ [\ln(1-D/I) - \ln(1-\theta_H)] / [\ln(1-\theta_L) - \ln(1-\theta_H)], & \text{if } \sigma = 1. \end{cases}$$

The numerator in the first expression is strictly increasing (strictly decreasing) in  $I$  and the denominator is strictly positive (strictly negative) when  $\sigma < 1$  ( $\sigma > 1$ ). Thus the first expression is strictly increasing in  $I$ , and the same is true for the second expression. Hence for all  $\sigma$ ,  $(1-\gamma)(1-p_D^*)$  is strictly decreasing in  $I$  for a given level of debt  $D$  such that  $D \in (R_L(I), R_S^*(I))$ .<sup>16</sup> Finally, the bounds of this interval are themselves strictly increasing in  $I$  (see Figure 2), as part (i) of Lemma 1 suggests.  $\square$

<sup>16</sup>Since in the above expression  $p_D^*$  depends on  $I$  and  $D$  only through their ratio, this result also follows from Claim 1.

The results of Lemma 1 and Claim 2 are illustrated on Figure 2.

To summarize, this section shows that our model generates reasonable comparative statics of the bankruptcy rate with respect to debt and income levels: a borrower with a lower income and/or higher debt is more likely to end up in bankruptcy.

## 5 Application: Government Intervention in Debt Restructuring

In this section, we use the framework that we have developed to analyze the effects of government intervention in debt restructuring. We show that understanding the workings of the private sector restructuring is crucial for designing a successful intervention.

Consider, for instance, a government intervention in a form of a mortgage modification program that aims at lowering the foreclosure rate (which corresponds to the bankruptcy rate in our model).<sup>17</sup> One example of such a program is HAMP (Home Affordable Mortgage Program) introduced in the U.S. in 2009. We will analyze effects of a program of this sort through the lens of our model, and show that the program may have unintended consequences if its design is naive and ignores the effects on private debt restructuring.

Before we proceed, it is important to point out that in our model a government intervention is never Pareto improving (assuming that the government is subject to the same frictions as private lenders), because the equilibrium allocation is constrained Pareto efficient. In our analysis, we abstract from the reasons for the intervention, simply take it as exogenous, and focus on its effects.

Within our framework, we will assume that the government steps in if bankruptcy is initiated, that is, if private renegotiation has been unsuccessful (i.e., did not take place). To keep the analysis simple, we model the intervention as the government making an offer to a delinquent borrower with probability  $p_G$  to make a repayment  $R_G$ . If the borrower accepts the offer and makes the repayment, the repayment is transferred to the lender lender.

For simplicity, we restrict our attention to the debt overhang case where the laissez-faire outcome is screening. However, all of the results outlined below also hold in the general case, assuming that the lender performs (constrained) screening. In analyzing the government intervention, we focus on the effect of the policy on the bankruptcy (foreclosure) rate.

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<sup>17</sup>The motivation for the government intervention may come from trying to limit the deadweight loss arising from foreclosures and/or out of concern for spill-overs through depressed house prices or “broken windows,” etc.

## 5.1 Deterministic Intervention

We begin by characterizing the simplest case where the government intervention is deterministic, i.e.,  $p_G = 1$ . We will illustrate most of our results in this simple case, and then show that some additional insights can be obtained in the case of random intervention.

Notice first that if the repayment  $R_G$  offered by the government exceeds  $R_H$ , then the intervention is completely irrelevant, because no borrower will ever want to make such a repayment. Thus, we can view the case of  $R_G \geq R_H$  as the no-intervention benchmark.

Consider next what happens if  $R_G \leq R_L$ , i.e., if the government offers a repayment that is lower than the lender's offer to delinquent borrowers in absence of an intervention. Clearly, such an intervention constrains the lender because no borrower would accept a higher repayment knowing that she would be offered the more favorable  $R_G$  upon rejecting the lender's offer. Thus, the effect of the intervention in this case is similar to the effect of lowering the debt level to  $D = R_G \leq R_L$ : a pooling outcome is achieved (i.e., all borrowers repay  $R_G$ ) and the bankruptcy rate inevitably drops to zero. Thus, in this case, the government policy is (trivially) effective, as it prevents all bankruptcies in equilibrium.

Finally, consider the less trivial case of  $R_G \in (R_L, R_H)$ , where the repayment offered by the government exceeds the willingness to pay of the low-cost borrowers, but is acceptable to the high-cost borrowers. In this case, the government intervention only restricts the lender's ability to extract repayment from the high-cost borrowers.

Recall from Section 3.2 that when  $D \in (R_L, R_S^*)$ , the restriction that the repayment cannot exceed the face value of debt forces the lender to renegotiate more often than he would have under debt overhang, and thus reduces the bankruptcy rate. Since the lender's ability to extract repayment from the high-cost type is limited anyway, he can extract repayment from a higher fraction of the low-cost type without distorting the incentives of the high-cost type. By analogy, one might infer that the government intervention with  $R_G \in (R_L, R_H)$  would have a similar effect and reduce the bankruptcy rate. However, in what follows, we show that the restriction imposed on the lender by the government is in fact quite different from the one imposed by the face value of debt. Moreover, in some cases, the bankruptcy rate actually increases in response to the intervention.

Formally, when  $R_G \in (R_L, R_H)$  and  $p_G = 1$ , the lender's problem becomes

$$\max_{p \in [0,1]} \gamma \hat{R}_S(p) + (1 - \gamma)pR_L, \quad (8)$$

where  $\hat{R}_S(p)$  is given by

$$u(I - \hat{R}_S) = pu(I - R_L) + (1 - p)u(I - R_G). \quad (9)$$

Note that the problem is identical to the familiar (3) subject to (2), where  $R_H$  has been replaced by  $R_G$ . That is, the government intervention basically amounts to lowering the high-cost borrowers' willingness to repay,  $R_H$ . We denote the solution to problem (8)–(9) by  $\hat{p}$ .

Notice that in equilibrium no borrower actually makes the repayment offered by the government. The low-cost borrowers reject the government's offer because  $R_G$  exceeds their willingness to pay, and the high-cost borrowers never receive the offer in the first place, because the lender makes them an offer that they prefer to delinquency. Thus, all renegotiation is performed by the lender, and the equilibrium bankruptcy rate is  $(1 - \gamma)(1 - \hat{p})$ .

In order to understand the effects of the intervention, we will study comparative statics of  $\hat{p}$  with respect to  $R_G$ , keeping in mind that  $R_G \geq R_H$  corresponds to the laissez-faire case. We will then compare the bankruptcy rate obtained under  $R_G \in (R_L, R_H)$  with that under  $R_G = R_H$ .

To this end, consider the first order condition of the lender's problem (8)–(9). It can be written as

$$(1 - \gamma)R_L = \gamma \underbrace{\frac{u(I - R_L) - u(I - R_G)}{u'(I - \hat{R}_S(p; R_G))}}_{= \frac{d\hat{R}_S}{dp}}, \quad (10)$$

where  $\hat{R}_S(p; R_G)$  is defined by (9). The left-hand side of the above equation is the marginal benefit of increasing  $p$  — it corresponds to an increase in the lender's profits due to a higher total repayment from the low-cost borrowers (and is unaffected by  $R_G$ ). The right-hand side is the marginal cost of an increase in  $p$  — it reflects the fact that  $\hat{R}_S$  must be reduced as  $p$  increases to keep the incentive constraint 9 satisfied.

The rate at which  $\hat{R}_S$  can be “exchanged” for  $p$ ,  $d\hat{R}_S/dp$ , depends on  $R_G$  through two channels. First, as  $R_G$  falls, the high-cost borrowers' utility from the lottery increases, and thus a smaller increase in utility  $u(I - \hat{R}_S)$  is needed to keep (9) satisfied as  $p$  increases. This effect is reflected in the numerator of the right-hand side of (10) being increasing in  $R_G$ . The second effect, working in the opposite direction, comes from the fact that as  $R_G$  falls, so does  $\hat{R}_S$ , which lowers the marginal utility  $u'(I - \hat{R}_S)$ . This in turn increases the

rate at which an increase in  $u(I - \hat{R}_S)$  translates into a decrease in  $\hat{R}_S$ . This second effect is reflected in the denominator of the right-hand side of (10) being increasing in  $R_G$ .

Whether the marginal benefit of an increase in  $p$ ,  $\gamma d\hat{R}_S/dp$ , increases or decreases with  $R_G$  depends on which of the two effects dominates. Suppose, for example, that  $R_H$  is very close to  $I$ , and  $R_G$  decreases from  $R_H$  marginally. Since bankruptcy is arbitrarily costly for the high-cost borrowers, even a small probability of bankruptcy is enough to make delinquency unattractive for them, and to induce them to make the prescribed payment.<sup>18</sup> This implies that  $u'(I - \hat{R}_S(p; R_G))$  is very responsive to the change in  $R_G$ , so that the negative effect dominates and thus the probability of renegotiation decreases as  $R_G$  decreases. But as  $R_G$  falls close to  $R_L$ , the numerator on the right-hand side of (10) becomes small, and the benefit of increasing  $p$  becomes greater than the cost. Thus, for  $R_G$  close enough to  $R_L$ , the positive effect dominates, and the intervention causes the lender to choose pooling as the optimal contract, i.e.,  $\hat{p} = 1$ .

We have thus established that  $\hat{p}$  is generally non-monotone in  $R_G$ . Most interestingly,  $\hat{p}$  decreases, and thus the bankruptcy rate rises, in response to the government intervention if  $R_G (< R_H)$  is close enough to  $I$ . That is, the government intervention aimed at preventing foreclosures may actually lead to an increase in foreclosures in equilibrium.

Figure 3 demonstrates the non-monotonicity of the bankruptcy rate as a function of  $R_G$  when  $p_G = 1$  using in a numerical example. In these computations, we use the logarithmic utility function  $u(c) = \ln c$  and assume that the cost of bankruptcy is proportional to income, i.e.,  $v(I, \theta) = u(I(1 - \theta))$ .<sup>19</sup> The borrower's income  $I$  is set to 2.55, and we pick the cost parameters  $\theta_L$  and  $\theta_H$  such that  $R_L = 0.5$  and  $R_H = 2.5$ . The fraction of high-cost borrowers  $\gamma$  is set to 0.4. The solid and dashed lines on Figure 3 correspond to the bankruptcy rates with and without the government intervention, respectively. Since the government intervention is irrelevant when  $R_G = R_H$ , the two lines coincide at that point.

Two scenarios illustrated on Figure 3 are of particular interest. First, consider the case where  $R_G$  is greater than  $R_L$  but is sufficiently close to it. In this case, the government intervention is completely successful in preventing foreclosures, despite appearing irrelevant — the repayment offered by the government is greater than that offered by the lender, as the lender offers  $R_L$  to all borrowers. That is, in equilibrium the government is not actively involved in restructuring mortgages. Yet, absent the intervention, the bankruptcy rate

<sup>18</sup>This follows from the assumption that the utility function satisfies the Inada condition.

<sup>19</sup>Note that with the log utility, this is equivalent to a “stigma” cost of bankruptcy, that is, a fixed additive utility cost:  $v(I, \theta) = \ln(I) + \ln(1 - \theta)$ .

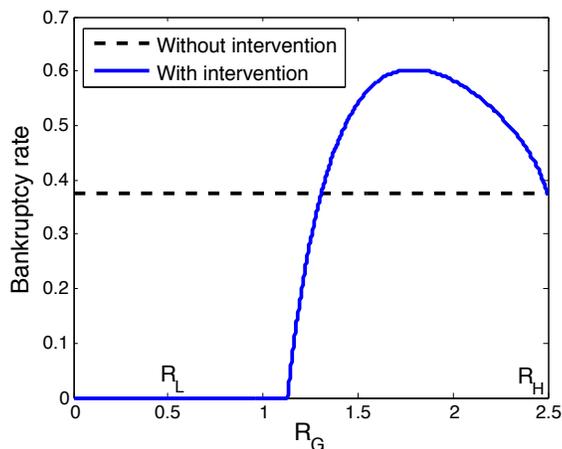


Figure 3: The bankruptcy (foreclosure) rate as a function of  $R_G$ , where  $p_G = 1$ . Parameter values:  $I = 2.55$ ,  $R_L = 0.5$ ,  $R_H = 2.5$ ,  $\gamma = 0.4$ .

would have been strictly positive (see the point  $R_G = R_H$ ).

Second, consider the case where  $R_G$  is below  $R_H$  but is sufficiently close to it. In this case, the government intervention “backfires” — it leads to an increase rather than a decrease in foreclosures. As we argued earlier, in this case the government offer is never accepted in equilibrium. In the next subsection we will demonstrate a case where the intervention backfires even though borrowers who receive the government offer, accept it.

## 5.2 Random Intervention

Next, we consider the case of a random intervention,  $p_G < 1$ , which can be interpreted as the borrower not being certain whether she is eligible for the government program. We will illustrate two additional scenarios that arise in this case that were absent in the case of the deterministic intervention.

In the first scenario, the policy is totally ineffective (i.e., it does not change the bankruptcy rate) although the government is busy preventing foreclosures. In the second scenario, the policy again backfires (leads to more foreclosures), but unlike in the case of the deterministic intervention, the government offer is accepted by some borrowers.

These scenarios are illustrated on Figure 4 using a numerical example similar to the one considered in the previous subsection. The probability of the government making an offer,  $p_G$ , is set to 0.3. Apart from the different  $p_G$ , the example depicted on the right panel of Figure 4 has all the same parameters as that on Figure 3. On the left panel, only the proportion of high-cost borrowers is different:  $\gamma = 0.1$ . Again, the point  $R_G = R_H$

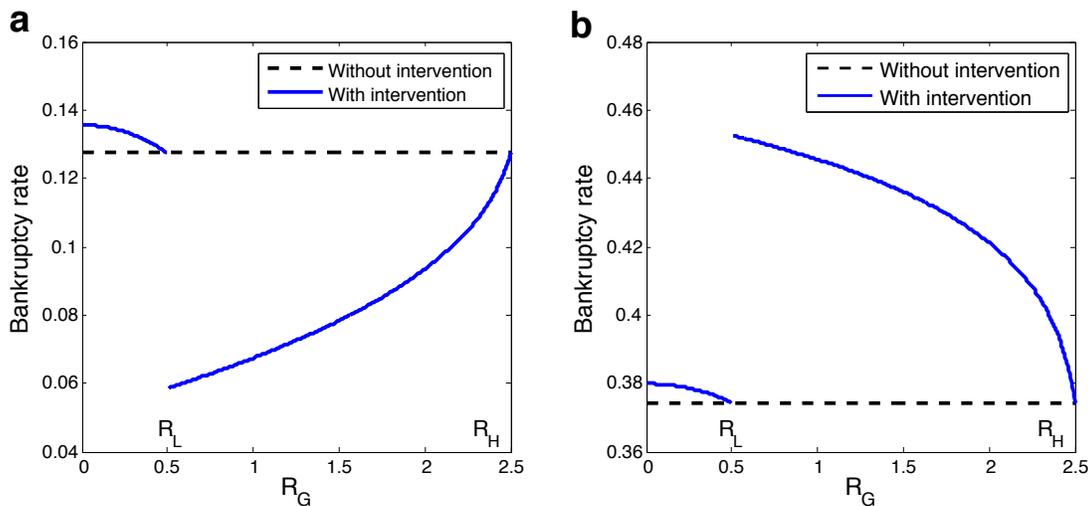


Figure 4: The bankruptcy (foreclosure) rate as a function of  $R_G$ , where  $p_G = 0.3$ . Parameter values:  $I = 2.55$ ,  $R_L = 0.5$ ,  $R_H = 2.5$ ; panel a:  $\gamma = 0.1$ , panel b:  $\gamma = 0.4$ .

corresponds to no intervention.

First, consider the case where  $R_G = R_L$  and  $p_G \leq p^*$  (which is the case on both panels of Figure 4). Without the intervention, the lender sets  $p$  equal to  $p^*$ . Anticipating the intervention, the lender simply adjusts the probability of renegotiation to offset the intervention, i.e.,  $\hat{p} + (1 - \hat{p})p_G = p^*$ . The resulting bankruptcy rate,  $(1 - \gamma)(1 - p^*)$ , is same as the laissez-faire one, and thus the intervention is ineffective. In this case, the government is busy preventing foreclosures, but its net effect is exactly nil.

Next, consider the case where  $R_G < R_L$ . Recall from the previous section that when  $p_G = 1$ , such an intervention always leads to pooling, i.e., reduces the bankruptcy rate to zero. This is not necessarily the case when  $p_G < 1$ . In fact, as Figure 4 shows, a random offer from the government with a repayment that is lower than that offered to delinquent borrowers by the lender ( $R_G < R_L$ ) can lead to an increase in the number of foreclosures.<sup>20</sup> In this case, the government program once again backfires. Compared to the case with deterministic intervention, now the government is actively participating in reducing foreclosures, as its offer (of a lower repayment) will be accepted in equilibrium by the delinquent borrowers. Yet, the foreclosure rate is higher than it would have been in the absence of the intervention.

<sup>20</sup>The argument behind this result is similar to the argument behind the non-monotonicity result described in the previous subsection, except now the government offer affects the lender's ability to extract repayment not only from the high-cost type, but also from the low-cost type.

Finally, when  $R_G \in (R_L, R_H)$ , the equilibrium bankruptcy rate can also be lower or higher than without intervention, as illustrated on the left and right panels of Figure 4, respectively. Recall that the model parameters for the two panels only differ in the proportion of high-cost borrowers,  $\gamma$ . When  $\gamma$  is higher, the lender has less incentives to renegotiate with delinquent (low-cost) borrowers, and so is more likely to behave adversely (i.e., renegotiate less often) in response to the intervention. Finally, notice that in the example on the right panel, an intervention with any offer  $R_G$  leads to the bankruptcy rate that is at least as high as in the laissez-faire case.

The results presented in this section indicate that explicit modeling of the private sector debt restructuring is key for analyzing the effects of a government intervention. In particular, the failure to understand how private lenders renegotiate with delinquent borrowers can lead to the policy having the opposite effect from the one intended.

## 6 Conclusions

We propose a simple model of consumer credit where a lender demands repayments from an indebted borrower, and the borrower's alternative to making a repayment is to declare bankruptcy. The main friction in the model is that the borrower's cost of bankruptcy is her private information.

We characterize the optimal contract in this environment. We show that the lender may choose to screen different types of borrowers using lotteries over repayments. The optimal screening contract has a natural economic interpretation as it generates three stages of default — delinquency, bankruptcy, and renegotiation. Specifically, the lender first offers a high repayment that only borrowers with the high bankruptcy cost accept. Borrowers with the low bankruptcy cost refuse to make this payment, and are thus considered delinquent. The lender then renegotiates by offering a lower repayment, but only with a fraction of the delinquent borrowers, while the rest end up in bankruptcy.

We show that the model generates reasonable comparative statics. In particular, the bankruptcy rate is decreasing in the debt level, and is increasing in the income level.

We apply the model to analyze the effects of a government intervention in debt restructuring, such as a mortgage modification program. We show that a program aiming to reduce foreclosures that overlooks the response of the private debt restructuring may lead to an increase rather than a reduction in the bankruptcy rate.

## Appendix: A Two-Period Model of Debt

This section presents a simple two-period model that endogenizes the debt level acquired in the first period, and has our basic mechanism at work in the second period.

Consider an environment with one borrower and several identical lenders. There are two periods,  $t = 1, 2$ . Assume for simplicity that the borrower's endowment in period 1 is zero. Her endowment in period 2, as well as her cost of declaring bankruptcy in that period, are random and unknown to everybody in period 1. Once the uncertainty is realized in period 2, the realization of the borrower's endowment is public knowledge, while her cost of bankruptcy is her private information.

We assume that the markets are incomplete: period 1 contracts cannot be made contingent on the realization of endowment (nor on the cost of bankruptcy) in period 2. Contracting in period 1 is restricted to specifying a transfer of resources to the borrower,  $c_1$ , and the face value of debt in period 2,  $D$ .<sup>21</sup> Lenders compete in contracts  $(c_1, D)$  that they offer to the borrower, and the borrower picks one contract or rejects all contracts (the latter option means living in autarky). In period 2, once uncertainty is realized, the borrower and the lender, whose contract the borrower accepted in period 1, interact in the environment described in the main text given the debt level  $D$  and the endowment realization  $I$ .

The borrower's preferences are represented by

$$u(c_1) + \beta E[(1 - \chi)u(I - R(D, I)) + \chi v(I, \theta)],$$

where  $R(D, I)$  is the repayment in period 2 and  $\chi$  is the indicator of bankruptcy. The expectation in the above expression is then taken over the endowment in the second period, the bankruptcy cost, and, if applicable, any contractual randomness.

The lenders have deep pockets and maximize  $-c_1 + E[R(D, I)]/(1 + r)$ , where  $r$  denotes the risk-free interest rate. Competing lenders earn zero profits in period 1, and thus  $(c_1, D)$  maximizes the borrower's expected utility in the first period subject to the lenders' zero expected profit condition. The equilibrium level of  $D$  thus solves the following problem:

$$\max_D u \left( \frac{1}{1 + r} E[R(D, I)] \right) + \beta E[(1 - \chi)u(I - R(D, I)) + \chi v(I, \theta)].$$

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<sup>21</sup>In other words,  $c_1$  is the amount borrowed and  $D/c_1$  is the gross interest rate. Note that the interest rate depends on the loan size.

Let  $D^*$  denote the solution to this problem. In this model, the type of contract offered by the lender in the second period will vary depending on the income realization. Given the equilibrium level of debt  $D^*$ , there is a threshold  $\bar{I}$  such that  $R_L(I) > D^*$  for all  $I > \bar{I}$ , so that the equilibrium in the corresponding states is constrained pooling, where all borrowers repay their debt. There is also a second threshold  $\underline{I} \in (0, \bar{I}]$  such that  $R_S^*(I) < D^*$  for all  $I < \bar{I}$ , so that the corresponding income states are characterized by debt overhang. In this case, the lender grants partial debt forgiveness to all borrowers, and then might renegotiate further with a fraction of low-cost borrowers who refuse to make the specified payment. For  $I \in (\underline{I}, \bar{I})$ , we have  $D^* \in (R_L(I), R_S^*(I))$ , which implies that the lender performs constrained screening. In this region, only the high-cost borrowers fully repay their debt, and the lender renegotiates with some low-cost borrowers. One can easily construct a distribution of period 2 incomes that results in all three cases occurring in equilibrium with positive probability. Thus, this extension illustrates that the mechanism highlighted in the paper applies if the level of debt is determined endogenously.

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