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On the Identification of Production Functions: How Heterogeneous is Productivity?

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Abstract

The estimation of production functions suffers from an unresolved identification problem caused by flexible inputs, such as intermediate inputs. We develop an identification strategy for production functions based on a transformation of the firm's short-run first order condition that solves the problem for both gross output and value-added production functions. We apply our approach to plant-level data from Colombia and Chile, and find that a gross output production function implies fundamentally different patterns of productivity heterogeneity than a value-added specification. This finding is consistent with our analysis of the bias induced by the use of value-added.

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1 Introduction

The identification and estimation of production functions using data on inputs and output is among the oldest empirical problems in economics. As first pointed out by Marschak and Andrews (1944), a key challenge for identification arises because a firm's productivity is transmitted to the firm's optimal choice of inputs, giving rise to an endogeneity issue known in the production function literature as the "transmission bias" (see e.g., Griliches and Mairesse, 1998). Early attempts to correct the transmission bias, i.e., using firm fixed effects or input prices as instruments, have proven to be both theoretically problematic and unsatisfactory in practice (see e.g., Griliches and Mairesse, 1998 and Ackerberg et al., 2007 for a review).

Two main approaches to the estimation of production functions have gained popularity: dynamic panel models (Arellano and Bond, 1991; Blundell and Bond, 1998, 2000) and structural estimation methods (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Ackerberg, Caves, and Frazer, 2006).[1] Both exploit instruments based on lagged input decisions of the firm as their source of identification.[2] However, as shown by Bond and Söderbom (2005) and Ackerberg, Caves, and Frazer (2006), the use of lagged input decisions as a source of identifying variation requires that *all* inputs in the production function be subject to adjustment frictions. For the capital input, a "time to build" restriction that causes capital to be fixed in the short term is a standard source of an adjustment friction. For the labor input, hiring/firing costs and search frictions are among the many possible candidate sources of an adjustment friction. We shall refer to inputs that are subject to adjustment frictions as *quasi-fixed* inputs.

[1] Cunha, Heckman, and Schennach (2010) present a different structural approach for the estimation of production functions under the assumption of a dynamic factor structure when noisy measures of the data are available.

[2] Lagged input decisions do not necessarily require that lagged values of the input be used as instruments. If the current value of an input was determined by a decision made before the current period, then the current value of the input can act as an instrument. We discuss this point in more detail in Section 2.

Our first contribution in this paper is to show that, while the use of lagged input decisions as instruments represents an important step towards identification of the production function, this solution nevertheless remains incomplete. In particular, this identification strategy fails in the presence of flexible inputs, i.e., inputs that are variable in each period and have no dynamic implications, which is how intermediate inputs (raw materials, energy, etc.) are typically modeled in empirical work. The reason is natural: since flexible inputs are not subject to adjustment frictions, the variation caused by these frictions is no longer available to identify flexible input elasticities. This lack of identifying variation for flexible inputs was forcefully pointed out by Bond and Söderbom (2005) and Ackerberg, Caves, and Frazer (2006). However, as we preview next, the resulting problems for empirical work are much more severe than has been previously appreciated.

In lieu of having credible instruments for intermediate inputs in the production function, a common alternative in the literature has been to subtract the value of intermediate inputs from gross output and redefine the object of interest to be a *value-added* production function. However, we show that even if the production function is Leontief in value added and intermediate inputs, which is the only plausible assumption under which value added can be both a theoretically *and* empirically well-defined object (see e.g., Parks, 1971, Berndt and Wood, 1975, and Denny and May, 1978), the value-added production function still suffers from the same identification problem caused by flexible inputs that confronts the gross output production function. The reason is that, even in the Leontief case, the ability to measure the theoretical notion of value added in the data further requires one of the remaining inputs (capital or labor) to be flexible. As a result, the identification problem associated with flexible inputs is simply transferred from intermediate inputs in the gross output production function to capital or labor in the value-added production function. Thus, regardless of whether one assumes a gross output or value-added production function, the use of lagged input decisions as instruments that has been the focus of the literature to date

breaks down, and the identification of the production function remains an open issue.

Our second key contribution is that we present a new identification strategy that solves the problem associated with flexible inputs in the production function. Our approach does not rely upon finding an instrument for flexible inputs or subtracting them from output, but is nevertheless consistent with the standard model of firm behavior used in the literature on structural estimation of production functions (see e.g., Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Bond and Söderbom, 2005; Ackerberg, Caves, and Frazer, 2006; Ackerberg et al., 2007). In particular, our identification strategy is based on a transformation of the firm's first order condition for flexible inputs. This transformation enables for the non-parametric regression of the observed revenue share of a flexible input against all observed inputs to non-parametrically identify both the flexible input's elasticity of production as well as the ex-post shocks to output. We shall refer to this non-parametric regression as the share regression.

The intuition for the identifying power of the share regression can be seen as follows. Index number methods, which equate input cost or revenue shares with input elasticities, have long been used to non-parametrically recover input elasticities in the absence of ex-post shocks (see e.g., Caves, Christensen, and Diewert, 1982).[3] However, ex-post shocks, which capture unanticipated productivity shocks to the firm and/or measurement error to the econometrician, are central to econometric models of the production function (see e.g., Griliches and Mairesse, 1998). Ackerberg, Caves, and Frazer (2006) show that the ex-post shock can be non-parametrically identified by the non-parametric regression of output on inputs: the shock equals the portion of output left non-parametrically unexplained by the in-

[3]Index number methods are grounded in three important economic assumptions. First, all inputs are flexible and competitively chosen, which rules out quasi-fixed inputs. Second, the production technology exhibits constant returns to scale, which while not strictly necessary is typically assumed in order to avoid imputing a rental price of capital. Third, there are no ex-post shocks to output, which can only be relaxed by assuming that elasticities are constant across firms, i.e., by imposing the parametric structure of Cobb-Douglas.

puts. Our share regression exploits the benefits of both approaches: we non-parametrically regress a revenue share on inputs rather than output on inputs. This enables us to non-parametrically identify *both* the flexible input elasticity of output *and* the ex-post shock. We further show that these two ingredients can be combined with the standard adjustment frictions on the remaining quasi-fixed inputs in order to identify and estimate the production function.

Our approach enables identification of both gross output and value-added specifications of the production function. However, since the value-added specification of the production function requires fairly specialized assumptions, a natural concern is that it may lead to misleading inferences. We show that for the standard model with Hicks-neutral technical change, using a value-added specification generally leads to biased estimates that are likely to overstate productivity heterogeneity. Determining the precise pattern of this "value-added bias," however, is ultimately an empirical question, and a significant one, since recovering productivity at the firm level is critical to addressing a wide of range of economic policy issues.[4]

Our third contribution is that we apply our identification strategy to plant-level data from Colombia and Chile to study the underlying patterns of productivity under gross output compared to value-added specifications. We find that productivity differences become orders of magnitude smaller and sometimes even change sign when we analyze the data through the lens of gross output rather than value added. For example, the standard 90/10 productivity ratio taken among all manufacturing firms in Chile is roughly 9 under value added (meaning that the 90th percentile firm is 9 times more productive than the 10th per-

[4]There is a large literature that has generated several stylized facts about heterogeneity of productivity at the firm level. Among these are the general understanding that even narrowly defined industries exhibit "massive" unexplained productivity dispersion (Dhrymes, 1991; Bartelsman and Doms, 2000; Syverson, 2004a,b; Collard-Wexler, 2010; Fox and Smeets, 2011), and that productivity is closely related to other dimensions of firm-level heterogeneity, such as importing (Kasahara and Rodrigue, 2008), exporting (Bernard and Jensen, 1995, Bernard and Jensen, 1999, Bernard et al., 2003), wages (Baily, Hulten, and Campbell, 1992), etc. See Syverson (2011) for a review of this literature.

centile firm), whereas under our gross output estimates this ratio falls to 2. The differences for the 95/5 ratio are even more stark: value added implies a ratio of 20 whereas gross output yields a ratio of only 3. Moreover, these dispersion ratios exhibit a remarkable degree of stability across industries and across the two countries when measured via gross output, but exhibit much larger cross-industry and cross-country variance when measured via value added. We further show that value added mis-measures in an economically significant way the productivity premium of firms that export, firms that import, firms that advertise, and higher wage firms as compared to gross output. These findings are consistent with the nature of value-added bias and emphasize the empirical relevance of our identification strategy for gross output production functions.

Our empirical results suggest that the bias introduced from using value added is at least as important, if not more so, than the transmission bias itself that has been the main focus of the production function estimation literature to date. In the conclusion, we sketch one policy implication of this finding for the problem of measuring the misallocation of resources that has been considered in recent work (e.g., Hsieh and Klenow, 2009). Our results suggest that the use of gross output production functions would generate substantially smaller differences in the estimates of misallocation across developed and developing countries as compared to the value-added approach that dominates this literature. This example highlights the policy significance of the identification of gross output production functions and the bias of value added.

The rest of the paper is organized as follows. In Section 2 we explain the identification problem caused by flexible inputs, and we show the insufficiency of value added as a means of addressing this problem. In Section 3 we present our identification strategy that allows for the non-parametric identification of the ex-post shock and the output elasticities with respect to flexible inputs and allows for the identification of the production function. Section 4 characterizes the bias induced by the use of value added. In Section 5 we de-

scribe the Colombian and Chilean data and show the results comparing gross output to value added for productivity measurement. In particular, we show evidence of large differences in unobserved productivity heterogeneity suggested by value added relative to gross output. Section 6 concludes with an example of the policy relevance of our results.

2 The Identification Problem

2.1 The Model

In order to illustrate the identification problem involved in estimating production functions, we adopt the broad elements of the model of production used in Olley and Pakes (1996) and the ensuing literature on the estimation of production functions (e.g., Levinsohn and Petrin, 2003; Ackerberg, Caves, and Frazer, 2006). The model consists of three basic components: 1) the structure of the production function, 2) the evolution of productivity, and 3) the timing of input decisions.

We observe a panel consisting of firms $j = 1, \ldots, J$ over periods $t = 1, \ldots, T$.[5] The firm's labor, capital, and intermediate inputs will be denoted by (L_{jt}, K_{jt}, M_{jt}) respectively. The log values of the inputs will be denoted by (l_{jt}, k_{jt}, m_{jt}). The firm's log anticipated productivity level is $\omega_{jt} \in \mathbb{R}$, which enters the production function in a Hicks-neutral fashion. We refer to ω_{jt} as anticipated productivity as it is known to the firm at the start of period t.

The relationship between a firm j's input and output in period t is expressed as

$$Q_{jt} = F(L_{jt}, K_{jt}, M_{jt})e^{\omega_{jt}}, \tag{1}$$

$$Y_{jt} = Q_{jt}e^{\varepsilon_{jt}}, \tag{2}$$

[5]For notational simplicity we assume a balanced panel, but unbalanced panels caused by attrition can be addressed using a selection correction analogous to that of Olley and Pakes (1996).

where $F(\cdot)$ is the production function, Q_{jt} is the output anticipated by the firm for a given vector of inputs (L_{jt}, K_{jt}, M_{jt}), and Y_{jt} is the measured output that is observed by the econometrician. The difference between the firm's anticipated output Q_{jt} and the measured output Y_{jt} is caused by the presence of the additional shock ε_{jt}, which represents an unanticipated productivity shock (in contrast to the anticipated Hicks-neutral shock ω_{jt}) and/or measurement error. For expositional simplicity, we will refer to ε_{jt} as the ex-post shock, ω_{jt} as anticipated productivity, and the sum, $\omega_{jt} + \varepsilon_{jt}$, simply as productivity. Expressed in logs, (2) becomes

$$y_{jt} = f(L_{jt}, K_{jt}, M_{jt}) + \omega_{jt} + \varepsilon_{jt}, \tag{3}$$

where $f(\cdot) = \ln F(\cdot)$. The econometric problem is to identify $f(\cdot)$ and thus recover productivity $\omega_{jt} + \varepsilon_{jt} = y_{jt} - f(L_{jt}, K_{jt}, M_{jt})$ for each firm/period observation.

Firm j takes its log anticipated productivity level ω_{jt} as a state variable in period t. Anticipated productivity evolves according to a first order Markovian process. Thus it can be expressed as $\omega_{jt} = h(\omega_{jt-1}) + \eta_{jt}$, where the term η_{jt} represents a mean zero innovation to the firm's productivity that, by assumption, is orthogonal to the firm's information set at period $t-1$.

Capital and labor are quasi-fixed inputs. As discussed by Bond and Söderbom (2005) and Ackerberg, Caves, and Frazer (2006), the adjustment frictions on the quasi-fixed inputs are a source of variation that identifies their elasticities. Although there are a number of alternative ways to model these adjustment frictions, we employ the "adjustment lag" assumption that quasi-fixed inputs adjust such that the period t levels of capital and labor were chosen at or before $t-1$. These timing restrictions imply that capital and labor are state variables for the firm for the period t. This in turn implies the conditional moment restriction $E[\eta_{jt} \mid k_{jt}, l_{jt}] = 0$, i.e., that today's innovation in productivity is mean indepen-

dent of capital and labor. These are the key moments for identifying elasticities associated with capital and labor. We have adopted the adjustment lag approach for concreteness of exposition, but our analysis is amenable to the many alternative assumptions on the evolution of the quasi-fixed inputs that one can make.[6]

The intermediate inputs M_{jt} (raw materials, energy, etc.) are the only inputs that firm j can adjust in period t. Moreover, intermediate inputs are static inputs, in that they have no dynamic implications, i.e., their period t levels do not affect the firm's profit in future periods. We will refer to this combination of being a variable and static input as being a *flexible* input. For notational simplicity, we treat intermediate inputs M_{jt} as a single composite input consisting of all the flexible inputs.

2.2 The Identification Problem with Flexible Inputs

We now summarize the fundamental identification problem related to the flexible inputs. This problem was recognized by Marschak and Andrews (1944), and stated formally by Bond and Söderbom (2005) and Ackerberg, Caves, and Frazer (2006). We focus attention in the main body on the classic case studied by Marschak and Andrews, which is the standard setup in the literature: perfect competition in the input and output markets. The perfect competition case makes the identification problem caused by intermediate inputs particularly evident, but the same problem also arises under imperfect competition as explained in Appendix A1.

Let ρ_t denote the intermediate input price and P_t denote the output price facing all firms in period t. Thus $(L_{jt}, K_{jt}, \omega_{jt}, \rho_t, P_t)$ is the vector of state variables from the perspective of firm j. Since capital and labor are determined prior to period t and the choice of inter-

[6]For example, we could assume that capital and labor are chosen in period t subject to adjustment costs as in Bond and Söderbom (2005). Another alternative is to assume that capital and/or labor are chosen at some point between t and $t-1$ as in Ackerberg, Caves, and Frazer (2006). In both these cases, the appropriate moment condition involves lagged capital and labor, i.e., $E[\eta_{jt} \mid k_{jt-1}, l_{jt-1}] = 0$.

mediate inputs does not have any dynamic implications, the prices of capital and labor are not relevant for the choice of intermediate inputs M_{jt}. If we let $\epsilon = E\left(e^{\varepsilon_{jt}}\right)$, the firm's first order condition with respect to M_{jt} yields,

$$P_t F_M(L_{jt}, K_{jt}, M_{jt}) e^{\omega_{jt}} \epsilon = \rho_t. \tag{4}$$

Thus M_{jt} is an implicit function of the state $(L_{jt}, K_{jt}, \omega_{jt}, \rho_t, P_t)$, i.e.,

$$M_{jt} = \mathbb{M}(L_{jt}, K_{jt}, \omega_{jt}, \rho_t, P_t) = \mathbb{M}_t(L_{jt}, K_{jt}, \omega_{jt}). \tag{5}$$

Two important facts are brought to light by (5). The first fact is that M_{jt} is clearly an endogenous regressor in (3) since it is partly determined by the unobserved (to the econometrician) ω_{jt}. Hence identification of the production function $F(\cdot)$ requires at a minimum that we control for the endogeneity of the regressor M_{jt}. However, the second fact made clear by (5) is that there is no source of cross-sectional variation in M_{jt} other than the firm's remaining productive inputs $(L_{jt}, K_{jt}, \omega_{jt})$. That is, there does not exist any exclusion restriction that generates cross-sectional variation in the intermediate input from outside of the production function. Thus, to use the language of Bond and Söderbom (2005) and Ackerberg, Caves, and Frazer (2006), intermediate inputs M_{jt} are "collinear" with the other productive inputs $(L_{jt}, K_{jt}, \omega_{jt})$.[7] This presents a fundamental identification problem.

[7]For further discussion and particular parametric examples of the collinearity of intermediate inputs in the production function, see Bond and Söderbom (2005) and Ackerberg, Caves, and Frazer (2006). Note that input prices and output prices, insofar as they may appear heterogeneous in the data, may reflect quality differences (see e.g., Griliches and Mairesse, 1998). As such, these prices should be explicitly included when measuring inputs and output, which is implicitly carried out by the standard procedure of measuring inputs and output via industry-specific deflation of monetary values. Thus the perceived price variation may be neither exogenous nor necessarily even exist in a real economic sense after inputs and output have been properly measured. This relation between possible input and/or output price variation and heterogeneous quality is one reason for why attempts to use perceived price variation as instruments have performed so poorly in practice (see e.g., Ackerberg et al., 2007).

Wooldridge (2009) suggests pairing the "proxy variable" assumption of the Olley and Pakes (1996)/Levinsohn and Petrin (2003)/Ackerberg, Caves, and Frazer (2006) framework with the use of lagged values of the flexible inputs, M_{jt-1}, to instrument for the current value, M_{jt}, in the production function. However, it is clear that this alternative one-step estimator does not change the fundamental identification problem we raise. To see why, observe that the alternative estimator suggested by Wooldridge is based on splitting anticipated productivity, ω_{jt}, into the expected component, $h(\omega_{jt-1})$, and the innovation, η_{jt}:

$$y_{jt} = \alpha l_{jt} + \beta k_{jt} + \gamma m_{jt} + h\left(\underbrace{\iota_{t-1}^{-1}\left(l_{jt-1}, k_{jt-1}, \iota_{jt-1}\right)}_{\omega_{jt-1}}\right) + \eta_{jt} + \varepsilon_{jt}, \tag{6}$$

where ι is the proxy variable for productivity (either investment in physical capital or intermediate inputs). Since

$$m_{jt} = m_t\left(l_{jt}, k_{jt}, \omega_{jt}\right) = \tilde{m}_t\left(l_{jt}, k_{jt}, l_{jt-1}, k_{jt-1}, \iota_{jt-1}, \eta_{jt}\right), \tag{7}$$

we are left with precisely the same collinearity problem (now in terms of (6) and (7)) that was expressed by (5): the only sources of variation in m_{jt} other than η_{jt} (an additive unobservable in equation (6)) are $(l_{jt}, k_{jt}, l_{jt-1}, k_{jt-1}, \iota_{jt-1})$, which are exactly the other explanatory variables in the estimating equation (6). Therefore, just as before, there does exist any exclusion restriction that generates cross-sectional variation in the intermediate input from outside of the production function, and hence the collinearity problem remains.

2.3 Value Added and the Identification Problem

A common empirical approach that seemingly avoids the identification problem caused by intermediate inputs is to exclude them from the model and redefine the object of interest to be a value-added production function. We now show that although intermediate inputs

do not appear in the value-added production function, the production function generally remains subject to same the collinearity problem that confronts the gross output production function. The fundamental difficulty is due to the fact that the identification of the value-added production function requires *both* that a value-added production function theoretically exists and that it can be empirically measured. Conditions under which both are satisfied cause the collinearity problem to reappear.

To better understand this fact, we recall that, as discussed by Sims (1969) and Arrow (1972), the value-added production function is a theoretically meaningful concept only when 1) the underlying gross output production function takes a nested (i.e., weakly separable) form and 2) productivity enters in a *value-added augmenting* way, i.e.,

$$Y_{jt} = F\left[\mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}}, M_{jt}\right], \tag{8}$$

where, for conceptual clarity, we have not included the ex-post shock ε_{jt} in the model (but return to a discussion of its possible role below). The interpretation of (8) is that the "primary inputs" of labor L_{jt} and capital K_{jt} combine to form value added via the value-added production function $\mathcal{H}$:

$$VA_{jt} = \mathcal{H}\left(L_{jt}, K_{jt}\right) e^{\omega_{jt}}, \tag{9}$$

which then combines with intermediate inputs M_{jt} via F to form final output Y_{jt}.[8]

Whereas equations (8) and (9) provide the theoretical foundation for a value-added production function to exist, as pointed out by Arrow (1972), value added VA_{jt} is ultimately a latent concept and cannot, in general, be directly observed. As has been emphasized in the

[8] As Arrow (1985) writes, "Without the separability assumption, however, it is hard to assign any definite meaning to real value added, and probably the best thing to say is that the concept should not be used when capital and labor are not separable from materials in production." (p. 458)

literature (see e.g., Parks, 1971, Berndt and Wood, 1975, and Denny and May, 1977,1978), the only cases in which the output of the value-added production function (9) underlying (8) can be observed are when F takes one of two extreme possible forms: 1) perfect substitution between intermediate inputs and value added, and 2) perfect complementary between intermediate inputs and value added.[9]

In a special case of perfect substitution, (8) becomes

$$Y_{jt} = VA_{jt} + M_{jt}.$$

Thus the standard empirical measure of real value added, the difference between deflated output and deflated intermediate inputs $VA_{jt}^{E} \equiv Y_{jt} - M_{jt}$, equals the latent value added VA_{jt}. However, perfect substitution is an unreasonable description of a production process, as it implies that final output can be produced from intermediate inputs alone. Hence, the empirical literature on value-added production functions often appeals to the opposite extreme case of perfect complements, i.e., where F takes a Leontief form. A standard representation of the Leontief case is

$$F\left(L_{jt}, K_{jt}, M_{jt}\right) = a \min\left[\mathcal{H}\left(L_{jt}, K_{jt}\right) e^{\omega_{jt}}, \mathcal{C}\left(M_{jt}\right)\right], \tag{10}$$

where a is a constant and $\mathcal{C}(\cdot)$ is a monotone increasing and concave function.[10] The key property of Leontief that allows for VA_{jt} to be measured in the data is that firms optimally chose inputs such that,

$$\mathcal{H}\left(L_{jt}, K_{jt}\right) e^{\omega_{jt}} = \mathcal{C}\left(M_{jt}\right). \tag{11}$$

[9] While the accounting measure of value added generally can be related to the primary inputs, for example using the duality results in Bruno (1978) and Diewert (1978), only under these two extreme assumptions on the underlying technology does this relationship correspond to the value-added production function, equation (9).

[10] In Appendix B1 we show that the ex-post shock, ε_{jt}, can only be incorporated into the value-added model if it is assumed to consist of only measurement error, and not an ex-post productivity shock.

Combined with equation (10) this implies that,

$$Y_{jt} = a\mathcal{H}\left(L_{jt}, K_{jt}\right) e^{\omega_{jt}} = a\mathcal{C}\left(M_{jt}\right). \tag{12}$$

Notice though that the empirical measure of value added, $VA^E = Y - M$ will not, in general, correspond to the latent concept of value added, VA. As a result, VA^E cannot be used to measure VA. However, as is made clear by equation (12), and as recognized in a recent application by Collard-Wexler, Asker, and De Loecker (2011), one can use gross output Y, instead of VA^E, as a measure of VA. When the production function is Leontief and when equation (12) holds, Y is equivalent to a constant a multiplied by VA. Thus, we can measure the latent concept of value added in the data, through gross output. Since VA does not include intermediate inputs, the value-added production function given by does not suffer from the collinearity problem associated with intermediate inputs.

The assumption in equation (11), however, is inconsistent with the identification assumption that capital and labor are quasi-fixed. To see why, notice that in this case, firms either cannot adjust capital and labor in period t or can only do so with some positive adjustment cost. The key consequence is that firms may optimally choose to *not* equate value added with intermediate inputs, i.e., it may be optimal for the firm to hold onto a larger stock of value added than can be combined with M_{jt} if L_{jt} and K_{jt} are both costly (or impossible) to downwardly adjust.[11] Thus, it will not necessarily be the case that $\mathcal{H}\left(L_{jt}, K_{jt}\right) e^{\omega_{jt}} = \mathcal{C}\left(M_{jt}\right)$, and as a result, value added cannot be properly measured

[11] For example, suppose $\mathcal{C}\left(M_{jt}\right) = M_{jt}^{0.5}$. For simplicity, also suppose that capital and labor are fixed one period ahead, and therefore cannot be adjusted in the short-run. Following the setup in Section 2.1, marginal revenue with respect to intermediate inputs equals the derivative of $\mathcal{C}\left(M_{jt}\right)$ multiplied by a and by the output price P_t when $M_{jt}^{0.5} < \mathcal{H}\left(L_{jt}, K_{jt}\right) e^{\omega_{jt}}$. When $M_{jt}^{0.5} \geq \mathcal{H}\left(L_{jt}, K_{jt}\right) e^{\omega_{jt}}$ increasing M_{jt} does not increase output due to the Leontief structure, so marginal revenue is 0. Marginal cost equals the price of intermediate inputs ρ_t. The firm's optimal choice of M is therefore given by $M_{jt} = \left(\frac{P_t}{\rho_t} 0.5a\right)^2$, if $\left(\frac{P_t}{\rho_t} 0.5a\right) < \mathcal{H}\left(L_{jt}, K_{jt}\right) e^{\omega_{jt}}$. But when $\left(\frac{P_t}{\rho_t} 0.5a\right) > \mathcal{H}\left(L_{jt}, K_{jt}\right) e^{\omega_{jt}}$, the firm no longer finds it optimal to set $\mathcal{H}\left(L_{jt}, K_{jt}\right) e^{\omega_{jt}} = \mathcal{C}\left(M_{jt}\right)$, and prefers to hold onto excess value added.

empirically. That is, the second equality in equation (12) no longer necessarily holds, and we have that

$$Y_{jt} = a\mathcal{C}\left(M_{jt}\right) \leq a\mathcal{H}\left(L_{jt}, K_{jt}\right) e^{\omega_{jt}}.$$

Consequently, the key property of the Leontief specification that allows for value added to be properly measured no longer holds.[12]

Suppose instead that we alter the standard timing assumptions on the primary inputs and assume that one of the primary inputs is flexible. In this case, the firm will optimally choose to equate $\mathcal{H}\left(L_{jt}, K_{jt}\right) e^{\omega_{jt}}$ and $\mathcal{C}\left(M_{jt}\right)$. When both M_{jt} and one of $\left(L_{jt}, K_{jt}\right)$ is flexible, it is never optimal to have excess value added or excess intermediate inputs (if there is an excess of either, given the lack of dynamic considerations for flexible inputs, it is always more profitable to reduce the level of the input in excess). Hence the result in equation (12) follows.

Notice though that the Leontief assumption delivers the result in equation (12) *only* because we assumed that one of the primary inputs is flexible. However, the flexibility of a primary input causes the value-added production function (9) to suffer from the same collinearity problem that was discussed in Section 2.2, but in this case with respect to the now assumed flexible primary input. That is, the collinearity problem caused by intermediate inputs in the gross output production function has simply been transferred to one of the primary inputs in the value-added production function.

[12]Note that this problem cannot be solved by simply allowing for measurement error on the right-hand side. Letting $\Delta_{jt} \geq 0$ denote the difference between $\mathcal{H}\left(L_{jt}, K_{jt}\right) e^{\omega_{jt}}$ and $\mathcal{C}\left(M_{jt}\right)$, we have that: $Y_{jt} = a\mathcal{H}\left(L_{jt}, K_{jt}\right) e^{\omega_{jt}} - a\Delta_{jt}$, where recall that a is a positive constant. Observe that Δ_{jt} will always take either positive values or zero. Furthermore, the higher the amount of primary inputs (capital and labor) that the firm employs, the more likely it is that Δ_{jt} will be positive. Thus the error term Δ_{jt} will exhibit correlation with the primary inputs in the value-added production function and hence cannot be treated econometrically as measurement error. It will also be correlated with lagged input levels, and therefore lagged input levels cannot be used as instruments to control for Δ_{jt}. Put another way, introducing the error Δ_{jt} does not solve the mis-measurement problem as it creates separate endogeneity problems of its own. Note that we could have defined the difference Δ_{jt} to be multiplicative instead of additive. Our argument holds in either case.

3 Non-parametric Identification via First Order Conditions

As we have just shown, gross output production functions are subject to an identification problem caused by flexible inputs, i.e., the collinearity problem. Furthermore, conditions under which value-added production functions can be measured in the data in general lead to an analogous collinearity problem for one of the remaining inputs (capital or labor). Identification of the production function thus remains an open question. In light of this, we develop a new identification strategy for production functions that deals with both transmission bias and the collinearity problem. We present our approach using the general case of the gross output production function. In Appendix B2 we show that our strategy can be extended to solve the collinearity problem with respect to capital or labor in order to identify the value-added production function. For simplicity, we present the main result under the perfect competition case that was considered in Section 2.2. However, in Appendix A2, we show that the same approach, i.e., transforming the first order condition of the flexible input, can be employed in the imperfectly competitive case.[13]

The key idea behind our identification strategy is to transform the first order condition (4) and combine it with the definition of the ex-post production function (2). In particular, we multiply the LHS of (4) by $\frac{F(L_{jt},K_{jt},M_{jt})}{F(L_{jt},K_{jt},M_{jt})}$, use the definitions in equations (1) and (2), and multiply both sides of (4) by $\frac{M_{jt}}{P_tQ_{jt}}$, to obtain $\frac{F_M(L_{jt},K_{jt},M_{jt})M_{jt}\epsilon}{F(L_{jt},K_{jt},M_{jt})e^{\varepsilon_{jt}}} = \frac{\rho_t M_{jt}}{P_tY_{jt}}$. Defining S_{jt} denote the observed revenue share of intermediate inputs we can write

$$S_{jt} = \frac{F_M(L_{jt}, K_{jt}, M_{jt})M_{jt}\epsilon}{F(L_{jt}, K_{jt}, M_{jt})e^{\varepsilon_{jt}}}, \tag{13}$$

[13] In addition to solving the identification problem for coefficients on flexible inputs, we show that the revenue share of a flexible input also allows us to non-parametrically recover the pattern of industry markups over time, a new and potentially useful result.

or, in logs

$$s_{jt} = \ln\left(G\left(L_{jt}, K_{jt}, M_{jt}\right)\epsilon\right) - \varepsilon_{jt}, \tag{14}$$

where $G\left(L_{jt}, K_{jt}, M_{jt}\right)$ is the elasticity of anticipated production (1) with respect to the intermediate inputs M_{jt}, and ε_{jt} is the ex-post shock from the production function. We will refer to equation (14) as the share regression. Since the ex-post shock ε_{jt} is by assumption independent of the inputs (L_{jt}, K_{jt}, M_{jt}), the non-parametric regression of s_{jt} on (L_{jt}, K_{jt}, M_{jt}) identifies both $G\left(L_{jt}, K_{jt}, M_{jt}\right)\epsilon$ and ε_{jt}. From the now known ε_{jt}, we can recover $\epsilon = E\left(e^{\varepsilon_{jt}}\right)$ and hence $G\left(L_{jt}, K_{jt}, M_{jt}\right)$.

The non-parametric identification of the flexible input elasticities then enables us to identify a gross output production function. First notice that, from the share regression above, we recover the left hand side of

$$G\left(L_{jt}, K_{jt}, M_{jt}\right) = \frac{F_M(L_{jt}, K_{jt}, M_{jt})M_{jt}}{F(L_{jt}, K_{jt}, M_{jt})}.$$

This equation forms a partial differential equation on M_{jt} that we can rewrite as

$$\frac{1}{F(L_{jt}, K_{jt}, M_{jt})} dF(L_{jt}, K_{jt}, M_{jt}) = G\left(L_{jt}, K_{jt}, M_{jt}\right)\frac{dM_{jt}}{M_{jt}}.$$

The solution to this equation will take the form

$$F\left(L_{jt}, K_{jt}, M_{jt}\right) e^{\mathscr{C}(L_{jt}, K_{jt})} = e^{\int G(L_{jt}, K_{jt}, M_{jt})\frac{dM_{jt}}{M_{jt}}}. \tag{15}$$

Since the right hand side of (15) can be readily calculated, the only thing that remains to be recovered is the constant (with respect to L_{jt}, K_{jt}) of integration $\mathscr{C}\left(L_{jt}, K_{jt}\right)$.

In order to recover the constant of integration, and hence the production function, we use the restrictions implied by the Markov process for productivity. First, by combining

equations (15), (1) and the now known ε_{jt}, we define

$$\mathcal{Y}_{jt} \equiv \ln\left(\frac{Y_{jt}}{e^{\varepsilon_{jt}} e^{\int G(L_{jt},K_{jt},M_{jt})\frac{dM_{jt}}{M_{jt}}}}\right). \tag{16}$$

Since $\omega_{jt} = \mathcal{Y}_{jt} + \mathscr{C}\left(L_{jt}, K_{jt}\right)$, we can write

$$\mathcal{Y}_{jt} + \mathscr{C}\left(L_{jt}, K_{jt}\right) = h\left(\mathcal{Y}_{jt-1} + \mathscr{C}\left(L_{jt-1}, K_{jt-1}\right)\right) + \eta_{jt}. \tag{17}$$

It follows then that, given the assumptions on η_{jt} described in Section 2.1, the constant of integration, $\mathcal{C}\left(L_{jt}, K_{jt}\right)$,[14] can be recovered from the following nonparametric regression:

$$\mathcal{Y}_{jt} = -\mathscr{C}\left(L_{jt}, K_{jt}\right) + \tilde{h}\left(\mathcal{Y}_{jt-1}, L_{jt-1}, K_{jt-1}\right) + \eta_{jt}.$$

3.1 Estimation

In this section we show how one can obtain a straightforward non-parametric estimator of the production function by roughly following the steps used to prove identification. In order to do so, we use standard series estimators as defined by Chen (2007). We follow Chen and propose a finite-dimensional truncated linear series given by a complete polynomial of order r for the elasticity in equation (14). That is, we propose to use a polynomial (in logs):

$$G_r\left(L_{jt}, K_{jt}, M_{jt}\right)\epsilon = \sum_{r_l+r_k+r_m \leq r} \gamma^{\epsilon}_{r_l,r_k,r_m} l_{jt}^{r_l} k_{jt}^{r_k} m_{jt}^{r_m}, \text{ with } r_l, r_k, r_m \geq 0$$

[14]Notice that this identification argument does not use the information that the same function $\mathscr{C}()$ is also inside the mean of the Markov process, $h()$. Our estimation algorithm, describe below, makes full use of this information.

and we use the sum squared residuals $\sum_{jt} \varepsilon_{jt}^2$ as our objective function. For example, for a complete polynomial of degree two, our estimator would solve:

$$\min_{\gamma} \sum_{j,t} \left\{ s_{jt} - \ln \left(\begin{array}{c} \gamma_0^{\epsilon} + \gamma_l^{\epsilon} l_{jt} + \gamma_k^{\epsilon} k_{jt} + \gamma_m^{\epsilon} m_{jt} \\ + \gamma_{ll}^{\epsilon} l_{jt}^2 + \gamma_{kk}^{\epsilon} k_{jt}^2 + \gamma_{mm}^{\epsilon} m_{jt}^2 \\ + \gamma_{lk}^{\epsilon} l_{jt} k_{jt} + \gamma_{lm}^{\epsilon} l_{jt} m_{jt} + \gamma_{km}^{\epsilon} k_{jt} m_{jt} \end{array} \right) \right\}^2 .$$

The solution to this problem is thus an estimator $\hat{G}_r\left(L_{jt}, K_{jt}, M_{jt}\right)\epsilon$, i.e., the elasticity up to the constant ϵ, as well as an estimate $\hat{\varepsilon}_{jt}$ of the ex-post shocks to production.[15] Since we can estimate $\hat{\epsilon} = \sum_{j,t} e^{\hat{\varepsilon}_{jt}}$, we can recover $\gamma = \frac{\gamma^{\epsilon}}{\epsilon}$, the parameters that define $\hat{G}_r\left(L_{jt}, K_{jt}, M_{jt}\right)$ free of the constant.

Following our identification strategy, in the second step, we calculate the integral in (15). One advantage of the polynomial sieve estimator we selected is that the integral will have a closed form solution:

$$\mathcal{G}_r\left(L_{jt}, K_{jt}, M_{jt}\right) \equiv \int \frac{G_r\left(L_{jt}, K_{jt}, M_{jt}\right)}{M_{jt}} dM_{jt} = \sum_{r_l + r_k + r_m \leq r} \frac{\gamma_{r_l, r_k, r_m}}{r_m + 1} l_{jt}^{r_l} k_{jt}^{r_k} m_{jt}^{r_m + 1}.$$

For example, for our degree two estimator we would have

$$\hat{\mathcal{G}}_r\left(L_{jt}, K_{jt}, M_{jt}\right) = \left(\begin{array}{c} \gamma_0 + \gamma_k k_{jt} + \gamma_l l_{jt} + \frac{\gamma_m}{2} m_{jt} + \gamma_{ll} l_{jt}^2 + \gamma_{kk} k_{jt}^2 \\ + \frac{\gamma_{mm}}{3} m_{jt}^2 + \gamma_{lk} l_{jt} k_{jt} + \frac{\gamma_{lm}}{2} l_{jt} m_{jt} + \frac{\gamma_{km}}{2} k_{jt} m_{jt} \end{array} \right) m_{jt}.$$

With an estimate of ε_{jt} and of $\int \frac{G(L_{jt}, K_{jt}, M_{jt})}{M_{jt}} dM_{jt}$ in hand, we can form a sample

[15] As with all nonparametric sieve estimators, the number of terms in the series should increase with the number of observations. Under mild regularity conditions this estimators will be consistent and asymptotically normal for sieve M-estimators like the one we propose. See Chen (2007).

analogue of (16):

$$\hat{\mathcal{Y}}_{jt} \equiv \ln\left(\frac{Y_{jt}}{e^{\hat{\varepsilon}_{jt}} e^{\hat{\mathcal{G}}_r(L_{jt},K_{jt},M_{jt})}}\right).$$

In order to recover the constant of integration in (17), we also use a similar complete polynomial series estimator.[16] That is, we use

$$\mathscr{C}_\tau\left(L_{jt}, K_{jt}\right) = \sum_{\tau_l+\tau_k\leq\tau} \alpha_{\tau_l,\tau_k} l_{jt}^{\tau_l} k_{jt}^{\tau_k}, \text{ with } \tau_l, \tau_k > 0$$

for some degree τ (that increases with the data), to form

$$\omega_{jt}\left(\delta\right) = \hat{\mathcal{Y}}_{jt} + \mathscr{C}_\tau\left(L_{jt}, K_{jt}\right).$$

We then run a non-parametric regression of $\omega_{jt}\left(\alpha\right)$ on $\omega_{jt-1}\left(\alpha\right)$ to recover $\eta_{jt}\left(\alpha\right)$. We use the restrictions implied by the moment conditions

$$E[\eta_{jt} \mid k_{jt}, l_{jt}] = 0$$

to form a standard sieve GMM criterion function to recover α.

3.2 Relation to Literature on Structural Production Function Estimation

It is instructive to compare our empirical strategy with the literature on the structural estimation of production functions à la Olley and Pakes (1996); Levinsohn and Petrin (2003); Ackerberg, Caves, and Frazer (2006). In particular, the Ackerberg, Caves, and Frazer (2006) approach identifies the ex-post shock ε_{jt} from a first stage regression of output on

[16] As is well known, it is not possible to separately identify a constant in the production function from mean productivity, $E\left(\omega_{jt}\right)$. In our context this means that we normalize $\mathscr{C}\left(L_{jt}, K_{jt}\right)$ so that it contains no constant.

all the inputs and a proxy variable. A typical choice of this proxy variable is the intermediate input M_{jt}. Then in a second stage, having separated ε_{jt} from the production function, moment conditions with the innovation η_{jt} are used to identify and estimate the production function parameters.

The validity of the second stage moments depends critically on the inputs that are included in the second stage having a source of variation independent of the innovation to productivity η_{jt}. Bond and Söderbom (2005) and Ackerberg, Caves, and Frazer (2006) suggest strategies based on a quasi-fixed assumption on the inputs included in the production function, i.e., a presumption that inputs are non-flexible in some fashion.[17] This requires that they explicitly preclude the flexible inputs M_{jt} from being included in the estimation, i.e., it necessitates using value-added production functions.[18] However, as we discuss in Section 2.3, when all of the primary inputs are quasi-fixed, regardless of whether the production function is assumed to be Leontief, the value-added form of the production function is mis-specified.

In contrast to the proxy variable approach, we identify the ex-post shock ε_{jt} in a first stage using the non-parametric share regression (14) rather than a non-parametric proxy equation. In addition to ε_{jt}, our non-parametric first stage allows us to recover the output elasticity of the intermediate inputs ξ_{jt}. In this sense, the non-parametric share regression contains more information than the non-parametric proxy regression. It is this additional information that allows us to solve the identification problem caused by flexible inputs in

[17]This feature is also shared by the dynamic panel model approach to production function estimation (Arellano and Bond, 1991; Blundell and Bond, 1998, 2000).

[18]This is because if M_{jt} is used as a proxy variable it is by assumption perfectly flexible, and therefore there exists no independent source of variation in it beyond the other terms appearing in the production function. The one proxy variable strategy that is consistent with using gross output production functions is the use of investment as the proxy variable and the application of adjustment frictions to *all* inputs (including the intermediate inputs M_{jt}). This assumes away the collinearity problem caused by M_{jt} being a flexible input by assuming that firms have no inputs that can be flexibly adjusted in the short run. Furthermore, as pointed out by Levinsohn and Petrin (2003), the use of investment as a proxy variable may be problematic as many firms (about half in our data) have zero investment in any given year. This violates the strict monotonicity assumption required to implement the proxy variable approach to begin with.

the production function. Our identification strategy otherwise makes the same assumptions as these structural methods regarding the evolution of productivity and the quasi-fixed inputs. The key advantage of our approach is that it provides the first empirical strategy that identifies the production function in the presence of both flexible and quasi-fixed inputs.

4 Value-Added Bias

The identification strategy that we introduced in the previous section provides a solution to the collinearity problem inherent in *both* gross output and value-added production functions, which has previously stood as a barrier to their identification. However, as we discuss in Section 2.3, value-added production functions are properly specified only under fairly specialized assumptions. In addition to requiring a Leontief technology and a flexible primary input (and not allowing for an ex-post productivity shock), the model requires that productivity takes the form of *value-added augmenting* technical change. This assumption departs from the standard notion of productivity underlying (1) (i.e., Hicks-neutral technical change), which defines productivity as the ability of a firm to produce more gross output holding fixed *all* inputs (not just the primary inputs). A natural concern is therefore that the use of value added may lead to misleading inferences about the production technology and productivity. Nevertheless, it has become common practice to relate the empirical measure of value added to capital and labor as a means of recovering productivity. We now ask the question: what happens when one estimates the relationship between the empirical measure of value added and the primary inputs (excluding intermediate inputs) when the model of production follows the general gross output setup outlined in Section 2.1?

In order to see why value added might lead to biased estimates, observe that if we only control for the variation in *some* inputs (say capital and labor), part of the heterogeneity in output among firms will be due to variation in the excluded inputs (intermediate inputs), in

addition to productivity. Since intermediate input usage will still be correlated with productivity (see equation (5)), the observed variation in output will overstate the true degree of productivity heterogeneity. Furthermore, since intermediate input usage will also be correlated with the inputs that are controlled for (capital and labor), this will also lead to biased output elasticity estimates for these inputs.

In order to characterize value-added bias more formally, consider the generic production function $F\left(L_{jt}, K_{jt}, M_{jt}\right)$ from Section 2.1. Recall that real value added is defined empirically as the difference between the deflated value of gross output and the deflated value of intermediate inputs, $VA_{jt}^{E} = Y_{jt} - M_{jt}$. Defining S_{jt} to be the share of intermediate input expenditures in total output, we have

$$VA_{jt}^{E} = Y_{jt}\left(1 - S_{jt}\right) = F\left(L_{jt}, K_{jt}, M_{jt}\right) e^{\omega_{jt}+\varepsilon_{jt}}\left(1 - S_{jt}\right). \tag{18}$$

While equation (18) hints at the potential sources of the bias—M_{jt} is still on the right hand side and the share of intermediate inputs is now part of the residual—it does not provide a clear explanation. In order to be able to derive an expression for the bias, we take a first order approximation (in logs) to the production function. Defining $x_{jt} = \left(k_{jt}, l_{jt}\right)'$ and $\beta = \left(\alpha_k, \alpha_l\right)'$, we obtain:

$$va_{jt}^{E} = x_{jt}'\beta + \alpha_m m_{jt} + \omega_{jt} + \varepsilon_{jt} + \ln\left(1 - S_{jt}\right).$$

Now consider the exercise of regressing log value added on the so-called "primary inputs" of capital and labor

$$va_{jt}^{E} = x_{jt}'b + e_{jt}.$$

The resulting expression for b is given by:

$$b = \beta + \underbrace{E\left[x_{jt}x'_{jt}\right]^{-1} E\left[x_{jt}\omega_{jt}\right]}_{\text{Transmission Bias}} + \underbrace{E\left[x_{jt}x'_{jt}\right]^{-1} E\left[x_{jt}\left(\alpha_m m_{jt} + \ln\left(1 - S_{jt}\right)\right)\right]}_{\text{Value-Added Bias}} + \underbrace{E\left[x_{jt}x'_{jt}\right]^{-1} E\left[x_{jt}\varepsilon_{jt}\right]}_{=0}.$$

The bias in estimating the coefficients on the primary inputs thus consists of two components. The first is the classic transmission bias, which results from the correlation between productivity and the primary inputs. The second is the new source of bias that we identify, value-added bias, which results from the failure of the subtraction of intermediate inputs from gross output to fully control for the contribution of intermediate inputs to output. In order to isolate the effect of the value-added bias for productivity estimates, suppose that the transmission bias can be controlled for, so that it is zero. In this case, the bias in the (log) productivity one would recover from this value-added regression is:

$$\begin{aligned}
(e_{jt}) - (\omega_{jt} + \varepsilon_{jt}) &= -x_{jt}\left(\beta - b\right) + \alpha_m m_{jt} + \ln\left(1 - S_{jt}\right) \\
&= \alpha_m \zeta^m_{jt} + \zeta^S_{jt},
\end{aligned}$$

where ζ^m_{jt} is equal to the residual from a regression of intermediate inputs on capital and labor, and ζ^S_{jt} is equal to the residual from a regression of one minus the share of intermediate inputs on capital and labor.

Since conditional on capital and labor, firms with more productivity will demand more intermediate inputs (see equation (5)), the first component of the bias will be positively correlated with productivity and therefore will tend to overstate productivity differences. Without further assumptions on how the primary inputs are chosen (which directly impacts the share, S_{jt}), it is not possible to sign the second component of the bias. However, notice that the ex-post component of productivity, ε_{jt}, is trivially positively correlated with $\ln\left(1 - S_{jt}\right)$, which will tend to overstate productivity differences. When the share of intermediate inputs is constant across observations, for example when the production function

is Cobb-Douglas in intermediate inputs (ignoring the ex-post shocks), the second component of the bias will be zero, and the entire value-added bias will lead to overestimates of productivity heterogeneity. Ultimately, however, it is an empirical question as to the magnitude and direction of the value-added bias as a whole, which we now explore.

5 Data and Application

We estimate separate production functions for the five largest 3-digit manufacturing industries in both Colombia and Chile, which are Food Products (311), Textiles (321), Apparel (322), Wood Products (331), and Fabricated Metal Products (381). We also estimate an aggregate specification grouping all manufacturing together. We estimate the production function in two ways. First, using our approach from Section 3 we estimate a gross output production function using a complete degree two polynomial series for both the FOC and the integration constant in the production function as described in section 3. That is, we use

$$\begin{aligned} G_2\left(L_{jt}, K_{jt}, M_{jt}\right) &= \gamma_0 + \gamma_l l_{jt} + \gamma_k k_{jt} + \gamma_m m_{jt} \\ &\quad + \gamma_{ll} l_{jt}^2 + \gamma_{kk} k_{jt}^2 + \gamma_{mm} m_{jt}^2 \\ &\quad + \gamma_{lk} l_{jt} k_{jt} + \gamma_{lm} l_{jt} m_{jt} + \gamma_{km} k_{jt} m_{jt} \end{aligned}$$

to estimate the intermediate inputs elasticity and,

$$\mathscr{C}_2\left(L_{jt}, K_{jt}\right) = \alpha_l l_{jt} + \alpha_k k_{jt} + \alpha_{ll} l_{jt}^2 + \alpha_{lk} k_{jt}^2 + \alpha_{lk} l_{jt} k_{jt}$$

for the constant of integration. Putting all the elements together, the gross output production function we estimate is given by:

$$
\begin{aligned}
go_{jt} &= \left(\begin{array}{c} \gamma_0 + \gamma_k k_{jt} + \gamma_l l_{jt} + \frac{\gamma_m}{2} m_{jt} + \gamma_{ll} l_{jt}^2 + \gamma_{kk} k_{jt}^2 \\ + \frac{\gamma_{mm}}{3} m_{jt}^2 + \gamma_{lk} l_{jt} k_{jt} + \frac{\gamma_{lm}}{2} l_{jt} m_{jt} + \frac{\gamma_{km}}{2} k_{jt} m_{jt} \end{array} \right) m_{jt} \\
&\quad -\alpha_l l_{jt} - \alpha_k k_{jt} - \alpha_{ll} l_{jt}^2 - \alpha_{lk} k_{jt}^2 - \alpha_{lk} l_{jt} k_{jt} + \omega_{jt} + \varepsilon_{jt},
\end{aligned} \tag{19}
$$

since, from equation (15), it is immediate that $y_{jt} = \int G\left(L_{jt}, K_{jt}, M_{jt}\right) \frac{dM_{jt}}{M_{jt}} - \mathscr{C}\left(L_{jt}, K_{jt}\right)$. Second, we estimate a value-added specification using the commonly-applied method developed by Ackerberg, Caves, and Frazer (2006), which builds on Olley and Pakes (1996) and Levinsohn and Petrin (2003), using a translog approximation:

$$
va_{jt} = \beta_l l_{jt} + \beta_k k_{jt} + +\beta_{ll} l_{jt}^2 + \beta_{kk} k_{jt}^2 + \beta_{lk} l_{jt} k_{jt} + \nu_{jt} + \epsilon_{jt}. \tag{20}
$$

In Table 1 we report estimates of the average output elasticities for each input for both the value-added and gross output models. We also report the sum of the elasticities. Since our second-order approximation does not impose homogeneity of the production function, this is not strictly-speaking an estimate of returns to scale, but it has a similar interpretation. In every case the value-added model overestimates the sum of elasticities relative to gross output, with an average difference of 3% in Colombia and 6% in Chile.

Value added also recovers dramatically different patterns of productivity as compared to gross output. Following Olley and Pakes (1996), we define productivity (in levels) as the sum of the anticipated and unanticipated components: $e^{\omega_{jt}+\varepsilon_{jt}}$.[19] In Tables 2A and 2B,

[19] Since our interest is in analyzing productivity heterogeneity we conduct our analysis using productivity in levels. An alternative would be to measure productivity in logs. However, the log transformation is only a good approximation for measuring percentage differences in productivity across groups when these differences are small, which they are not in our data. We have also computed results based on log productivity. As expected, the magnitude of our results changes, however, our qualitative results comparing gross output

for Colombia and Chile respectively, we report estimates of several frequently analyzed statistics of the resulting productivity distributions. In the first three rows of each table we report ratios of percentiles of the productivity distribution, a commonly used measure of productivity dispersion. There are two important implications of these results. First, value added suggests a much larger amount of heterogeneity in productivity across plants *within* an industry, as the various percentile ratios are much smaller under gross output. For Colombia, the average 75/25, 90/10, and 95/5 ratios are 1.88, 3.69, and 6.14 under value added, and 1.30, 1.71, and 2.14 under gross output. For Chile, the average 75/25, 90/10, and 95/5 ratios are 2.76, 8.02, and 17.92 under value added, and 1.45, 2.13, and 2.85 under gross output. The value-added estimates imply that the 95th percentile plant produces more than 6 times more output in Colombia, and almost 18 times more output in Chile, than the 5th percentile plant using the same amount of inputs. In stark contrast, we find that under gross output, the 95th percentile plant produces only 2 times more output in Colombia, and 3 times more output in Chile, than the 5th percentile plant.

The second important result is that value added also implies much more heterogeneity *across* industries, which is captured by the finding that the range of the percentile ratios across industries are much tighter using the gross output measure of productivity. For example, for the 95/5 ratio, the value-added estimates indicate a range from 4.36 to 11.01 in Colombia and from 12.52 to 25.08 in Chile, whereas the gross output estimates indicate a range from 1.96 to 2.29 and from 2.39 to 3.17. The surprising aspect of these results is that the dispersion in productivity appears far more stable both across industries and across countries when measured via gross output as opposed to value added. In the conclusion we sketch some important policy implications of this finding for empirical work on the misallocation of resources.

In addition to having much larger overall amounts of productivity dispersion, results

and value added still hold.

based on value added also suggest a substantially different relationship between productivity and other dimensions of plant-level heterogeneity. We examine several commonly-studied relationships between productivity and other plant characteristics. In the last four rows of Tables 2A and 2B we report percentage differences in productivity based on whether plants export some of their output, import intermediate inputs, have positive advertising expenditures, and pay above the median (industry) level of wages.

Using the value-added estimates we find that, for most industries, exporters are found to be more productive than non-exporters, with exporters appearing to be 82% more productive in Colombia and 14% more productive in Chile across all industries. Once we account for intermediate inputs using the gross output specification, these estimates of productivity differences fall to 9% in Colombia and 1% in Chile, and actually turn negative (although not statistically different from zero) in some cases.

A similar pattern exists when looking at importers of intermediate inputs. In all but one case, importers appear much more productive than non-importers under value added. The average productivity difference is 13% in Colombia and 40% in Chile. However, under gross output, these numbers fall to 9% and 11% respectively. The same story holds for differences in productivity based on advertising expenditures. Moving from value added to gross output, the estimated difference in productivity drops for most industries in Colombia, and for all industries in Chile. In several cases it becomes statistically indistinguishable from zero.

Another striking contrast arises when we compare productivity between plants that pay wages above versus below the industry median. Using the productivity estimates from a value-added specification, firms that pay wages above the median industry wage are found to be substantially more productive, with the estimated differences ranging from 34%-63% in Colombia and from 47%-123% in Chile. In every case the estimates are statistically significant. Using the gross output specification, these estimates fall to 10%-22% in Colombia

and 18%-28% in Chile, representing a fall by a factor of 3, on average, in both countries.

In order to isolate the importance of the value-added bias relative to the transmission bias, in Tables 3 and 4 we repeat the above analysis without correcting for the endogeneity of inputs. We examine the raw effects in the data by estimating productivity using simple OLS to estimate equations (19) and (20). As can be seen from Tables 3A and 3B, the general pattern of results, that value added overstates productivity differences across many dimensions, is remarkably similar to our previous results both qualitatively and quantitatively.

While the results in Table 3 may suggest that transmission bias is not empirically important, in Table 4 we show evidence to the contrary. In particular, we report the average input elasticities based on estimates for the gross output model using OLS and using our method to correct for transmission bias. A well-known result is that failing to control for the transmission bias leads to overestimates of the coefficients on more flexible inputs. The intuition behind this is that the more flexible the input is, the more it responds to productivity shocks and the higher the degree of correlation between that input and unobserved productivity. The estimates in Table 4 show that the OLS results substantially overestimate the output elasticity of intermediate inputs in every case. The average difference is 28%, which illustrates the importance of controlling for the endogeneity generated by the correlation between input decisions and productivity.

An important implication of our results is that, while controlling for transmission bias certainly has an effect, the bias induced from using value added has a much larger effect on the productivity estimates than the transmission bias in the gross output production function. This suggests that avoiding value-added bias may be more important from a policy perspective than controlling for the transmission bias that has been the primary focus in the production function literature. Our approach avoids value-added bias by allowing for the use of gross output production functions while simultaneously correcting for the

transmission bias.

6 Conclusion

In this paper we show that the classic problem of identification of production functions in the presence of quasi-fixed and flexible inputs has remained an unresolved issue. We offer a new identification strategy that closes this loop. The key to our approach is exploiting the non-parametric information contained in the first order condition for the flexible inputs.

Our empirical analysis demonstrates that value added can generate substantially biased patterns of productivity heterogeneity as compared to gross output, which suggests that empirical studies of productivity based on value added may lead to fundamentally misleading policy implications. To illustrate this possibility, consider the recent literature that uses productivity dispersion to explain cross-country differences in output per worker through resource misallocation. As an example, the recent influential paper by Hsieh and Klenow (2009) finds substantial heterogeneity in productivity dispersion (defined as the variance of log productivity) across countries as measured using value added. In particular, when they compare the United States with China and India, the variance of log productivity ranges from 0.40-0.55 for China and 0.45-0.48 for India, but only from 0.17-0.24 for the United States. They then use this estimated dispersion to measure the degree of misallocation of resources in the respective economies. In their main counterfactual they find that by reducing the degree of misallocation in China and India to that of the United States, aggregate TFP would increase by 30%-50% in China and 40%-60% in India. In our datasets for Colombia and Chile, the corresponding estimates of the variance in log productivity using a value-added specification are 0.43 and 0.94, respectively. Thus their analysis applied to our data would suggest that there is similar room for improvement in aggregate TFP in Colombia, and much more in Chile.

However, when productivity is measured using our gross output framework, our empirical findings suggest a much different result. The variance of log productivity using gross output is 0.08 in Colombia and 0.14 in Chile. These significantly smaller dispersion measures could imply that there is much less room for improvement in aggregate productivity for Colombia and Chile. Since the 90/10 ratios we obtain for Colombia and Chile using gross output are quantitatively very similar to the estimates obtained by Syverson (2004b) for the United States (who also employed gross output but in an index number framework), this also suggests that the degree of differences in misallocation of resources between developed and developing countries may not be as large as the analysis of Hsieh and Klenow (2009) implies.[20]

Exploring the role of gross output production functions for policy problems such as the one above could be a fruitful direction for future research. A key message of this paper is that insights derived under value added could significantly bias policy conclusions, and the use of gross output production functions is thus possibly critical for policy analysis. Our identification strategy provides researchers with a stronger foundation for using gross output production functions in practice.

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[20]Hsieh and Klenow note that their estimate of log productivity dispersion for the United States is larger than previous estimates by Foster, Haltiwanger, and Syverson (2008) by a factor of almost 4. They attribute this to the fact that Foster, Haltiwanger, and Syverson use a selected set of homogeneous industries. However, another important difference is that Foster, Haltiwanger, and Syverson use gross output measures of productivity rather than value-added measures. Given our results in Section 5, it is likely that a large part of this difference is due to Hsieh and Klenow's use of value added, rather than their selection of industries.

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Appendix A: Revenue Production Functions

Appendix A1

In this appendix we illustrate how our approach can be extended to accommodate imperfect competition. Since firms no longer necessarily charge the same price, when output prices are not observed, deflated revenue no longer properly measures the quantity that the firm produces. As a result, unobserved variation in firm-specific prices needs to be addressed in the production function. One solution to this problem suggested by Klette and Griliches (1996) and recently applied by De Loecker (2007) is to model unobserved prices via an iso-elastic demand system. While this demand system decomposes the problem of unobserved prices in the production function in a convenient way, we now show that exactly the same identification problem involving intermediate inputs arises in the resulting *revenue* production function. Furthermore, as the solution involves modeling unobserved output prices,

value added is no longer an appropriate measure of output as consumers have demand for gross output and not for value added.

Suppose we follow Klette and Griliches (1996) and De Loecker (2007) and specify an iso-elastic demand system derived from an underlying representative CES utility function,

$$\frac{P_{jt}}{\Pi_t} = \left(\frac{Q_{jt}}{Q_t}\right)^{\frac{1}{\sigma}} e^{\chi_{jt}}, \tag{21}$$

where P_{jt} is the output price of the firm, Π_t is the industry price index, Q_t is a quantity index that plays the role of an aggregate demand shifter, χ_{jt} is an observable (to the firm) demand shock, and σ is the assumed constant elasticity of demand.

What we observe in the data is the firm's real revenue, which in logs is given by $r_{jt} = (p_{jt} - \pi_t) + y_{jt}$. Recalling equations (1) and (2), and replacing (21) into the log revenue equation gives

$$r_{jt} = \left(1 + \frac{1}{\sigma}\right) f(L_{jt}, K_{jt}, M_{jt}) - \frac{1}{\sigma} q_t + \chi_{jt} + \left(1 + \frac{1}{\sigma}\right) \omega_{jt} + \varepsilon_{jt}. \tag{22}$$

Thus, the anticipated part of the residual is a linear combination of the demand shock and productivity shock, i.e., $\chi_{jt} + \left(1 + \frac{1}{\sigma}\right) \omega_{jt}$. However, it is precisely this same linear combination of the demand and productivity shocks that shifts the intermediate input demand. To see why, observe that short-run profits are given by

$$\begin{aligned} \text{SRProfits}_{jt} &= P_{jt} Q_{jt} - \rho_t M_{jt} \\ &= \Pi_t \left(\frac{1}{Q_t}\right)^{\frac{1}{\sigma}} \left(F\left(L_{jt}, K_{jt}, M_{jt}\right)\right)^{1+\frac{1}{\sigma}} e^{\chi_{jt} + \left(1+\frac{1}{\sigma}\right)\omega_{jt}} - \rho_t M_{jt}. \end{aligned}$$

Notice that the productivity and demand shocks (ω_{jt}, χ_{jt}) enter profits only through the sum, $\chi_{jt} + \left(1 + \frac{1}{\sigma}\right) \omega_{jt}$. It is only this linear combination that matters for short-run profits

and hence for any static optimization problems, including the demand for intermediate inputs M_{jt}, i.e., $M_{jt} = \mathbb{M}_t(L_{jt}, K_{jt}, \chi_{jt} + \left(1 + \frac{1}{\sigma}\right)\omega_{jt})$.[21] Thus we are left with precisely the same identification problem that was shown in Section 2. Even though we now have two unobservables (ω_{jt}, χ_{jt}), there still does not exist any exclusion restriction that can vary intermediate inputs M_{jt} from outside of the revenue production function (22).

Appendix A2

We now show that our empirical strategy can be extended to the setting with imperfect competition and revenue production functions such that 1) we solve the identification problem with flexible inputs and 2) we can recover time-varying industry markups.[22] In fact, our empirical strategy allows for the identification of pieces of the production function as well as the time pattern (but not the level) of markups without having to specify any particular demand system.

Letting Λ_{jt} denote a firm's marginal cost, the first order condition with respect to M_{jt} for a cost minimizing firm is: $\Lambda_{jt} F_M\left(L_{jt}, K_{jt}, M_{jt}\right) e^{\omega_{jt}} = \rho_t$. Following the same strategy as before, we can rewrite this expression in terms of the observed log revenue share, which becomes

$$s_{jt} = -\psi_{jt} + \ln G\left(L_{jt}, K_{jt}, M_{jt}\right)\epsilon - \varepsilon_{jt}, \tag{23}$$

where $\psi_{jt} = \ln \frac{P_{jt}}{\Lambda_{jt}}$ is the log of the markup, $G(\cdot)$ is the output elasticity of intermediate inputs, and ε_{jt} is the ex-post shock. Equation (23) nests the one obtained for the perfectly competitive case in (14), the only difference being the addition of the log markup ψ_{jt} which is equal to 0 under perfect competition. The two key differences between the perfectly com-

[21] As opposed to the flexible inputs, it is not clear how the demand for quasi-fixed inputs (e.g., capital) will depend on ω_{jt} and χ_{jt}, i.e., whether it will depend on the same linear combination or on each component independently (and whether it will be monotone in each shock).

[22] This stands in contrast to the Klette and Griliches (1996) approach that can only allow for a markup that is time-invariant.

petitive case and this case are that 1) we no longer restrict the firm's price to be constant, and 2) the firm's anticipated revenue share no longer equals the input elasticity directly, but rather it equals the input elasticity divided by the markup charged by the firm.

We now show how to use the share regression (23) to identify production functions among imperfectly competitive firms. As opposed to the Klette and Griliches (1996) setup, in which markups are restricted to be constant, $\psi_{jt} = \psi$, we allow for markups to change over time, $\psi_{jt} = \psi_t$. In this case (23) becomes

$$s_{jt} = -\psi_t + \ln\left(G\left(L_{jt}, K_{jt}, M_{jt}\right)\epsilon\right) - \varepsilon_{jt}. \tag{24}$$

The intermediate input elasticity can be rewritten so that we can break it into two parts: a component that varies with inputs and a constant μ, i.e., $\ln \xi_{jt} = \ln G\left(L_{jt}, K_{jt}, M_{jt}\right) = \ln G^{\mu}\left(L_{jt}, K_{jt}, M_{jt}\right) + \mu$. Then, equation (24) becomes

$$\begin{aligned} s_{jt} &= (-\psi_t + \mu) + \epsilon + \ln G^{\mu}\left(L_{jt}, K_{jt}, M_{jt}\right) - \varepsilon_{jt} \\ &= -\delta_t + \epsilon + \ln G^{\mu}\left(L_{jt}, K_{jt}, M_{jt}\right) - \varepsilon_{jt}. \end{aligned} \tag{25}$$

As equation (25) makes clear, without having to specify a demand system, we can non-parametrically recover the ex-post shock ε_{jt} (and hence ϵ), the output elasticity of intermediate inputs up to a constant $\ln \xi_{jt}^{\mu} = \ln G^{\mu}(L_{jt}, K_{jt}, M_{jt}) = \ln G(L_{jt}, K_{jt}, M_{jt}) - \mu$, and the time-varying markups up to the same constant, $\delta_t = \psi_t - \mu$. This is, to the best of our knowledge, a new result.[23] Recovering the growth pattern of markups over time is

[23] In contrast to the results in Hall (1988) and Basu and Fernald (1995, 1997), which are based on index number methods that allow them to recover a firm and time invariant markup, we recover the growth pattern of markups but not the level. However, we do not need to impose the restriction that all inputs are flexibly and competitively chosen, impose restrictions on the shape of the production function (e.g., homogeneity), or compute/estimate the rental rate of capital/profit for the entrepreneur. As we show below, we can recover the level of markups with the addition of standard restrictions on product demand.

useful as an independent result as it can, for example, be used to check whether market power has increased over time, or to analyze the behavior of market power with respect to the business cycle.

As before, we can correct our estimates for ϵ and solve the differential equation that arises from equation (25). However, because we can still only identify the elasticity up to the constant μ, we have to be careful about keeping track of it as we can only calculate $\int G^{\mu}\left(L_{jt}, K_{jt}, M_{jt}\right) \frac{dM_{jt}}{M_{jt}} = e^{-\mu} \int G\left(L_{jt}, K_{jt}, M_{jt}\right) \frac{dM_{jt}}{M_{jt}}$. It follows that

$$f\left(L_{jt}, K_{jt}, M_{jt}\right) e^{-\mu} + \mathscr{C}\left(L_{jt}, K_{jt}\right) e^{-\mu} = \int G^{\mu}\left(L_{jt}, K_{jt}, M_{jt}\right) \frac{dM_{jt}}{M_{jt}}.$$

From this equation it is immediately apparent that, without further information, we will not be able to separate the integration constant $\mathscr{C}\left(L_{jt}, K_{jt}\right)$ from the unknown constant μ.

To see how one can recover both the constant μ and the constant of integration, we specify a generalized version of the demand system in equation (21)

$$\frac{P_{jt}}{\Pi_t} = \left(\frac{Q_{jt}}{Q_t}\right)^{\frac{1}{\sigma_t}} e^{\chi_{jt}}, \tag{26}$$

where we allow for time-varying markups and hence $\psi_t = -\ln\left(1 + \frac{1}{\sigma_t}\right)$.[24] In this case the observed log-revenue production function (22) becomes

$$r_{jt} = \left(1 + \frac{1}{\sigma_t}\right) f(L_{jt}, K_{jt}, M_{jt}) - \frac{1}{\sigma_t} q_t + \chi_{jt} + \left(1 + \frac{1}{\sigma_t}\right) \omega_{jt} + \varepsilon_{jt}. \tag{27}$$

However, we can write $\left(1 + \frac{1}{\sigma_t}\right) = e^{-\gamma_t} e^{-\mu}$. We know γ_t from our analysis above, so only

[24] We can also allow for time-varying firm-specific markups. If we let $\Upsilon_{jt} > 0$ be an independent demand shock that is realized after inputs are chosen, then *expected markups* will be equalized across firms, i.e., $E\left(\Psi_{jt}\right) = \Psi_t$ and χ_{jt} will enter into the firm's period t input decisions. That is, while actual markups $\Psi_{jt} = \frac{P_{jt}}{\Lambda_{jt}}$ will be firm specific due to the Υ_{jt} demand shocks, firms will still have ex-ante symmetric markups.

μ is unknown. Replacing back into (27) we get

$$\begin{aligned} r_{jt} &= e^{-\gamma_t} e^{-\mu} q_{jt} - \left(e^{-\gamma_t} e^{-\mu} - 1\right) q_t + \chi_{jt} + \varepsilon_{jt} \\ &= e^{-\gamma_t} e^{-\mu} f\left(K_{jt}, L_{jt}, M_{jt}\right) - \left(e^{-\gamma_t} e^{-\mu} - 1\right) q_t + \left[\left(e^{-\gamma_t} e^{-\mu}\right) \omega_{jt} + \chi_{jt}\right] + \varepsilon_{jt} \end{aligned} \tag{28}$$

We then follow a similar strategy as before. As in equation (16) we first form an observable variable

$$\mathcal{R}_{jt} \equiv \ln\left(\frac{P_{jt} Y_{jt}}{e^{\varepsilon_{jt}} e^{e^{-\gamma_t} \int G^{\mu}(L_{jt}, K_{jt}, M_{jt}) \frac{dM_{jt}}{M_{jt}}}}\right),$$

where we now use revenues (the measure of output we observe), include $e^{-\gamma_t}$, as well as using$G^{\mu}\left(\right)$ instead of the now unobservable $G\left(\right)$. Replacing into (28) we obtain

$$\mathcal{R}_{jt} = - \quad e^{-\gamma_t - \mu} \mathscr{C}\left(L_{jt}, K_{jt}\right) - \left(e^{-\gamma_t} e^{-\mu} - 1\right) q_t + \left[\left(e^{-\gamma_t} e^{-\mu}\right) \omega_{jt} + \chi_{jt}\right].$$

From this equation it is clear that the constant μ will be identified from variation in the observed demand shifter q_t. Without having recovered γ_t from the share regression first, it would not be possible to identify time-varying markups. Note that in equation (27) both σ_t and q_t change with time and hence q_t cannot be used to identify σ_t unless we restrict $\sigma_t = \sigma$ as in Klette and Griliches (1996) and De Loecker (2007).

Finally, we can only recover a linear combination of productivity and the demand shock, $\left(1 + \frac{1}{\sigma_t}\right) \omega_{jt} + \chi_{jt}$. The reason is obvious: since we do not observe prices, we have no way of disentangling whether, after controlling for inputs, a firm has higher revenues because it is more productive (ω_{jt}) or because it can sell at a higher price (χ_{jt}). Since we can write $\omega_{jt}^{\mu} = \left(1 + \frac{1}{\sigma_t}\right) \omega_{jt} + \chi_{jt}$ as a function of the parts that remain to be recovered

$$\omega_{jt}^{\mu} = \mathcal{R}_{jt} + e^{-\gamma_t - \mu} \mathscr{C}\left(L_{jt}, K_{jt}\right) + \left(e^{-\gamma_t} e^{-\mu} - 1\right) q_t$$

by imposing the Markovian assumption on this combination, $\omega_{jt}^{\mu} = h\left(\omega_{jt-1}^{\mu}\right) + \eta_{jt}$, we can use similar moment restrictions as before, $E\left(\eta_{jt}|k_{jt}, l_{jt}\right) = 0$, to identify the constant of integration $\mathscr{C}\left(L_{jt}, K_{jt}\right)$ as well as μ (and hence the level of the markups).

Appendix B: Value Added under Leontief

Appendix B1: Incorporating the Ex-Post Shock in Value Added:

We consider the two interpretations of the ex-post shock separately. First, suppose that ε_{jt} consists of only ex-post productivity shocks. In order for the weak-separability assumption necessary for the theoretical concept of value added to be well-defined, ε_{jt} must enter the production function modifying only value added: $Y_{jt} = a \min\left[\mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}+\varepsilon_{jt}}, \mathcal{C}\left(M_{jt}\right)\right]$. However, in this case, the equality between $\mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}+\varepsilon_{jt}}$ and $\mathcal{C}\left(M_{jt}\right)$ (or between $\mathcal{H}\left(L_{jt}, K_{jt}\right)e^{\omega_{jt}}$ and $\mathcal{C}\left(M_{jt}\right)$) no longer results from the firm's short run optimality condition, equation (12) no longer holds, and the model is mis-specified. Hence ε_{jt} cannot include ex-post productivity shocks.

Alternatively, suppose that ε_{jt} consists of only measurement error, so that output net of measurement error is given by $Q_{jt} = a \min\left[\mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}}, \mathcal{C}\left(M_{jt}\right)\right]$. Since measurement error does not affect the firm's problem, the equality between between $\mathcal{H}\left(L_{jt}, K_{jt}\right)e^{\omega_{jt}}$ and $\mathcal{C}\left(M_{jt}\right)$ still holds. In this case we have the following expression for the production function:

$$Y_{jt} = a\mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}+\varepsilon_{jt}}. \tag{29}$$

Appendix B2: The Non-Parametric Share Regression Approach with Value Added:

Recall the assumptions necessary to justify an empirical value-added form of the production function: 1) the gross output production function is Leontief in value added and intermediate inputs, 2) either capital or labor is flexible, 3) productivity is value-added augment-

ing, and 4) the ex-post shock consists only of measurement error. In this case, equation (29) holds and the value-added model is free of mis-specification. As we show in Section 2.3, in this case the value-added model is subject to the collinearity problem associated with the flexible primary input (capital or labor).

We now show how to adapt our identification strategy in Section 3 so that it can be used to identify the value-added model and solve the collinearity problem. In this case the first order condition with respect to the flexible primary input, e.g., labor, is given by:

$$P_t a \mathcal{H}_L(L_{jt}, K_{jt}) e^{\omega_{jt}} - \rho_t \frac{dM_{jt}}{dL_{jt}} = w_t, \tag{30}$$

where the second term accounts for changes in intermediate inputs with respect to a change in labor, w_t is the wage rate, and $\mathcal{H}_L$ is the derivative of the value-added production function with respect to labor. From the optimality condition for intermediate inputs, $\mathcal{C}(M_{jt}) = \mathcal{H}(L_{jt}, K_{jt}) e^{\omega_{jt}}$, we have $\mathcal{C}'(M_{jt})\, dM_{jt} = \mathcal{H}_L(L_{jt}, K_{jt}) e^{\omega_{jt}} dL_{jt}$, hence we can rewrite (30) as

$$\frac{\mathcal{H}_L(L_{jt}, K_{jt}) L_{jt}}{\mathcal{H}(L_{jt}, K_{jt})} \left(1 - \frac{\rho_t}{P_t} \frac{1}{a\mathcal{C}'(M_{jt})}\right) = \frac{w_t L_{jt}}{P_t Q_{jt}}.$$

This equation can be further rewritten to derive a share regression analogous to equation (14):

$$s_{jt}^L - \ln\left(1 - \frac{\rho_t}{P_t} \frac{1}{a\mathcal{C}'(M_{jt})}\right) = g^{VA}(L_{jt}, K_{jt}) - \varepsilon_{jt}, \tag{31}$$

where s_{jt}^L is the log of the ratio of the wage bill to gross output as measured in the data, and $g^{VA}(\cdot)$ is the log of the value-added elasticity of labor, i.e., the elasticity of $VA_{jt} = \mathcal{H}(L_{jt}, K_{jt}) e^{\omega_{jt}}$ with respect to labor.

If a and $\mathcal{C}'(M_{jt})$ were known, one could use equation (31) to identify the elasticity of the flexible primary input (labor). While it is unlikely that $a\mathcal{C}'(M_{jt})$ is known a-priori, we now show that it can in fact be recovered from the data. To see why, notice that, since

$\mathcal{C}\left(M_{jt}\right) = \mathcal{H}(L_{jt}, K_{jt})e^{\omega_{jt}}$, it follows that

$$Q_{jt} = a\mathcal{C}\left(M_{jt}\right).$$

Hence $a\mathcal{C}\left(M_{jt}\right)$ (and $a\mathcal{C}'\left(M_{jt}\right)$) can be identified from a non-parametric regression of log gross output on M_{jt}. Since we can now form the left hand side of (31) (and from it recover the elasticity and the ex-post shocks), we can then combine it with equation (29) to identify the production function following the same steps described in Section 3.

Table 1: Average Input Elasticities of Output

(Structural Estimates: Value Added vs. Gross Ouput)

Colombia

	Industry (ISIC Code)											
	311		**321**		**322**		**331**		**381**		**All**	
	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)
Labor	0.70	0.21	0.65	0.31	0.83	0.39	0.86	0.42	0.89	0.42	0.78	0.35
	(0.04)	(0.01)	(0.06)	(0.03)	(0.03)	(0.02)	(0.06)	(0.04)	(0.04)	(0.02)	(0.01)	(0.01)
Capital	0.33	0.09	0.36	0.14	0.16	0.06	0.12	0.02	0.25	0.09	0.31	0.13
	(0.02)	(0.01)	(0.04)	(0.02)	(0.02)	(0.01)	(0.04)	(0.03)	(0.03)	(0.01)	(0.01)	(0.00)
Intermediates	--	0.71	--	0.56	--	0.53	--	0.53	--	0.54	--	0.56
		(0.01)		(0.01)		(0.01)		(0.01)		(0.01)		(0.00)
Sum	1.03	1.01	1.01	1.01	0.99	0.98	0.98	0.96	1.14	1.06	1.09	1.04
	(0.03)	(0.01)	(0.04)	(0.02)	(0.02)	(0.01)	(0.06)	(0.04)	(0.02)	(0.01)	(0.01)	(0.00)

Chile

Labor	0.77	0.27	0.93	0.42	0.95	0.44	0.92	0.38	0.96	0.48	0.77	0.35
	(0.02)	(0.01)	(0.04)	(0.02)	(0.04)	(0.02)	(0.04)	(0.02)	(0.04)	(0.02)	(0.01)	(0.01)
Capital	0.33	0.10	0.24	0.10	0.20	0.06	0.19	0.06	0.25	0.13	0.37	0.15
	(0.01)	(0.00)	(0.02)	(0.01)	(0.03)	(0.01)	(0.02)	(0.01)	(0.02)	(0.01)	(0.01)	(0.00)
Intermediates	--	0.69	--	0.57	--	0.58	--	0.62	--	0.53	--	0.59
		(0.00)		(0.01)		(0.01)		(0.01)		(0.01)		(0.00)
Sum	1.10	1.06	1.17	1.09	1.14	1.08	1.11	1.06	1.22	1.13	1.13	1.08
	(0.02)	(0.01)	(0.03)	(0.02)	(0.03)	(0.01)	(0.03)	(0.01)	(0.03)	(0.01)	(0.01)	(0.00)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers in the first column are based on a value-added specification of a translog approximation to the production function and are estimated using the method from Ackerberg, Caves, and Frazer (2006). The numbers in the second column are based on a gross output specification estimated using our approach.

c. Since the input elasticities are heterogeneous across firms, we report the average input elasticities within each given industry.

Table 2A: Heterogeneity in Productivity

(Structural Estimates)

Colombia

	Industry (ISIC Code)											
	311		**321**		**322**		**331**		**381**		**All**	
	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)
75/25 ratio	2.20	1.27	1.97	1.34	1.66	1.27	1.73	1.31	1.78	1.31	1.95	1.35
	(0.06)	(0.02)	(0.09)	(0.03)	(0.03)	(0.01)	(0.08)	(0.05)	(0.04)	(0.02)	(0.02)	(0.01)
90/10 ratio	5.17	1.60	3.71	1.78	2.87	1.63	3.08	1.74	3.33	1.73	4.01	1.81
	(0.27)	(0.03)	(0.30)	(0.06)	(0.09)	(0.03)	(0.37)	(0.18)	(0.13)	(0.03)	(0.07)	(0.01)
95/5 ratio	11.01	1.96	6.36	2.29	4.36	1.99	4.58	2.23	5.31	2.13	6.86	2.27
	(1.10)	(0.05)	(0.75)	(0.14)	(0.22)	(0.05)	(0.97)	(0.35)	(0.34)	(0.06)	(0.17)	(0.02)
Exporter	3.62	0.11	0.20	0.07	0.16	0.06	0.26	0.18	0.20	0.09	0.51	0.07
	(0.99)	(0.03)	(0.10)	(0.03)	(0.07)	(0.03)	(0.65)	(0.35)	(0.05)	(0.02)	(0.12)	(0.01)
Importer	-0.25	0.04	0.27	0.10	0.29	0.13	0.06	0.06	0.26	0.10	0.20	0.11
	(0.08)	(0.02)	(0.10)	(0.03)	(0.08)	(0.03)	(0.56)	(0.27)	(0.06)	(0.02)	(0.05)	(0.01)
Advertiser	-0.46	-0.04	0.20	0.07	0.13	0.04	0.02	0.05	0.15	0.06	-0.13	0.03
	(0.10)	(0.02)	(0.07)	(0.03)	(0.04)	(0.02)	(0.10)	(0.06)	(0.04)	(0.02)	(0.06)	(0.01)
Wages > Median	0.59	0.10	0.60	0.22	0.41	0.18	0.34	0.16	0.55	0.22	0.63	0.20
	(0.19)	(0.02)	(0.09)	(0.03)	(0.03)	(0.02)	(0.17)	(0.09)	(0.06)	(0.02)	(0.05)	(0.01)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers in the first column are based on a value-added specification of a translog approximation to the production function and are estimated using the method from Ackerberg, Caves, and Frazer (2006). The numbers in the second column are based on a gross output specification estimated using our approach.

c. In the first three rows we report ratios of productivity for plants at various percentiles of the productivity distribution. In the remaining four rows we report estimates of the productivity differences between plants (as a fraction) based on whether they have exported some of their output, imported intermediate inputs, spent money on advertising, and paid wages above the industry median. For example, in industry 311 value added implies that a firm that advertises is, on average, 46% less productive than a firm that does not advertise.

Table 2B: Heterogeneity in Productivity

(Structural Estimates)

Chile

	Industry (ISIC Code)											
	311		**321**		**322**		**331**		**381**		**All**	
	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)
75/25 ratio	2.92	1.37	2.56	1.45	2.58	1.41	3.06	1.48	2.45	1.51	3.00	1.51
	(0.05)	(0.01)	(0.07)	(0.02)	(0.07)	(0.02)	(0.08)	(0.01)	(0.06)	(0.02)	(0.03)	(0.01)
90/10 ratio	9.02	1.86	6.77	2.09	6.76	2.05	10.12	2.25	6.27	2.28	9.19	2.30
	(0.30)	(0.02)	(0.30)	(0.05)	(0.33)	(0.04)	(0.60)	(0.05)	(0.27)	(0.04)	(0.15)	(0.02)
95/5 ratio	21.29	2.39	13.56	2.80	14.21	2.70	25.08	3.00	12.52	3.06	20.90	3.17
	(0.99)	(0.04)	(0.85)	(0.08)	(0.77)	(0.08)	(2.05)	(0.10)	(0.78)	(0.09)	(0.48)	(0.03)
Exporter	0.27	0.01	0.07	0.01	0.18	0.07	0.12	-0.02	0.03	-0.01	0.20	0.01
	(0.10)	(0.02)	(0.07)	(0.03)	(0.08)	(0.03)	(0.12)	(0.02)	(0.06)	(0.03)	(0.04)	(0.01)
Importer	0.71	0.12	0.22	0.08	0.31	0.12	0.44	0.14	0.30	0.09	0.46	0.14
	(0.11)	(0.02)	(0.05)	(0.02)	(0.05)	(0.02)	(0.10)	(0.03)	(0.05)	(0.02)	(0.03)	(0.01)
Advertiser	0.18	0.04	0.09	0.03	0.15	0.05	0.04	0.03	0.07	0.01	0.14	0.06
	(0.05)	(0.01)	(0.04)	(0.02)	(0.04)	(0.02)	(0.04)	(0.01)	(0.04)	(0.02)	(0.02)	(0.01)
Wages > Median	1.23	0.21	0.47	0.18	0.62	0.21	0.68	0.21	0.56	0.20	0.99	0.28
	(0.09)	(0.01)	(0.05)	(0.02)	(0.06)	(0.02)	(0.08)	(0.02)	(0.06)	(0.02)	(0.04)	(0.01)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers in the first column are based on a value-added specification of a translog approximation to the production function and are estimated using the method from Ackerberg, Caves, and Frazer (2006). The numbers in the second column are based on a gross output specification estimated using our approach.

c. In the first three rows we report ratios of productivity for plants at various percentiles of the productivity distribution. In the remaining four rows we report estimates of the productivity differences between plants (as a fraction) based on whether they have exported some of their output, imported intermediate inputs, spent money on advertising, and paid wages above the industry median. For example, in industry 311 value added implies that a firm that advertises is, on average, 18% more productive than a firm that does not advertise.

Table 3A: Heterogeneity in Productivity

(Uncorrected OLS Estimates)

Colombia

	Industry (ISIC Code)											
	311		**321**		**322**		**331**		**381**		**All**	
	Value Added (OLS)	Gross Output (OLS)	Value Added (OLS)	Gross Output (OLS)	Value Added (OLS)	Gross Output (OLS)	Value Added (OLS)	Gross Output (OLS)	Value Added (OLS)	Gross Output (OLS)	Value Added (OLS)	Gross Output (OLS)
75/25 ratio	2.17	1.16	1.86	1.21	1.65	1.17	1.72	1.23	1.78	1.23	1.93	1.24
	(0.06)	(0.01)	(0.06)	(0.01)	(0.03)	(0.01)	(0.06)	(0.02)	(0.04)	(0.01)	(0.02)	(0.00)
90/10 ratio	5.15	1.42	3.50	1.51	2.81	1.44	3.05	1.57	3.30	1.53	3.96	1.58
	(0.26)	(0.02)	(0.18)	(0.04)	(0.08)	(0.02)	(0.22)	(0.06)	(0.12)	(0.02)	(0.06)	(0.01)
95/5 ratio	10.86	1.74	5.77	1.82	4.23	1.74	4.67	2.01	5.22	1.82	6.81	1.94
	(0.97)	(0.04)	(0.56)	(0.08)	(0.20)	(0.04)	(0.66)	(0.15)	(0.32)	(0.04)	(0.15)	(0.02)
Exporter	3.42	0.09	-0.03	-0.01	0.10	0.00	0.21	0.10	0.12	0.03	0.45	0.01
	(0.93)	(0.03)	(0.04)	(0.01)	(0.05)	(0.01)	(0.21)	(0.10)	(0.04)	(0.02)	(0.12)	(0.01)
Importer	-0.23	-0.02	0.09	0.00	0.21	0.02	0.02	-0.03	0.20	0.05	0.14	0.04
	(0.07)	(0.01)	(0.06)	(0.01)	(0.06)	(0.01)	(0.06)	(0.02)	(0.05)	(0.01)	(0.04)	(0.01)
Advertiser	-0.46	-0.07	0.11	-0.04	0.10	-0.03	0.01	-0.02	0.08	0.00	-0.16	-0.02
	(0.10)	(0.03)	(0.05)	(0.02)	(0.03)	(0.01)	(0.07)	(0.03)	(0.04)	(0.01)	(0.05)	(0.01)
Wages > Median	0.51	0.06	0.49	0.10	0.39	0.13	0.33	0.11	0.50	0.13	0.56	0.13
	(0.16)	(0.02)	(0.07)	(0.02)	(0.03)	(0.01)	(0.08)	(0.03)	(0.04)	(0.01)	(0.05)	(0.01)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers in the first column are based on a value-added specification of a translog aproximation to the production function and are estimated using OLS. The numbers in the second column are based on a gross output specification estimated using OLS.

c. In the first three rows we report ratios of productivity for plants at various percentiles of the productivity distribution. In the remaining four rows we report estimates of the productivity differences between plants (as a fractio based on whether they have exported some of their output, imported intermediate inputs, spent money on advertising, and paid wages above the industry median. For example, in industry 311 value added implies that a firm that advertises is, on average, 46% less productive than a firm that does not advertise.

Table 3B: Heterogeneity in Productivity

(Uncorrected OLS Estimates)

Chile

	Industry (ISIC Code)											
	311		**321**		**322**		**331**		**381**		**All**	
	Value Added (OLS)	Gross Output (OLS)	Value Added (OLS)	Gross Output (OLS)	Value Added (OLS)	Gross Output (OLS)	Value Added (OLS)	Gross Output (OLS)	Value Added (OLS)	Gross Output (OLS)	Value Added (OLS)	Gross Output (OLS)
75/25 ratio	2.91	1.30	2.57	1.40	2.56	1.36	3.07	1.39	2.47	1.46	3.01	1.45
	(0.05)	(0.00)	(0.07)	(0.01)	(0.07)	(0.01)	(0.08)	(0.01)	(0.06)	(0.01)	(0.03)	(0.00)
90/10 ratio	9.00	1.72	6.63	1.97	6.64	1.91	10.21	2.03	6.27	2.14	9.13	2.14
	(0.29)	(0.01)	(0.31)	(0.04)	(0.29)	(0.03)	(0.57)	(0.04)	(0.26)	(0.04)	(0.15)	(0.01)
95/5 ratio	20.93	2.15	13.49	2.57	14.20	2.45	25.26	2.77	12.18	2.80	20.64	2.86
	(0.96)	(0.02)	(0.83)	(0.07)	(0.80)	(0.05)	(2.05)	(0.07)	(0.77)	(0.06)	(0.47)	(0.03)
Exporter	0.17	-0.01	0.04	-0.02	0.12	0.01	0.12	-0.02	0.00	0.00	0.15	-0.01
	(0.09)	(0.02)	(0.06)	(0.02)	(0.08)	(0.02)	(0.09)	(0.02)	(0.06)	(0.02)	(0.04)	(0.01)
Importer	0.57	0.03	0.20	0.04	0.26	0.06	0.41	0.07	0.27	0.06	0.41	0.09
	(0.09)	(0.01)	(0.04)	(0.02)	(0.05)	(0.01)	(0.09)	(0.03)	(0.05)	(0.02)	(0.03)	(0.01)
Advertiser	0.12	0.00	0.07	0.01	0.11	0.02	0.02	0.01	0.05	0.01	0.10	0.04
	(0.04)	(0.01)	(0.04)	(0.02)	(0.04)	(0.01)	(0.04)	(0.01)	(0.04)	(0.02)	(0.02)	(0.01)
Wages > Median	1.11	0.12	0.45	0.15	0.58	0.16	0.66	0.13	0.53	0.16	0.94	0.24
	(0.07)	(0.01)	(0.05)	(0.02)	(0.06)	(0.02)	(0.07)	(0.02)	(0.06)	(0.02)	(0.03)	(0.01)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers in the first column are based on a value-added specification of a translog aproximation to the production function and are estimated using OLS. The numbers in the second column are based on a gross output specification estimated using OLS.

c. In the first three rows we report ratios of productivity for plants at various percentiles of the productivity distribution. In the remaining four rows we report estimates of the productivity differences between plants (as a fraction) based on whether they have exported some of their output, imported intermediate inputs, spent money on advertising, and paid wages above the industry median. For example, in industry 311 value added implies that a firm that advertises is, on average, 12% more productive than a firm that does not advertise.

Table 4: Average Input Elasticities of Output

(Gross Output: Structural vs. Uncorrected OLS Estimates)

Colombia

	Industry (ISIC Code)											
	311		**321**		**322**		**331**		**381**		**All**	
	Gross Output (OLS)	Gross Output (GNR)	Gross Output (OLS)	Gross Output (GNR)	Gross Output (OLS)	Gross Output (GNR)	Gross Output (OLS)	Gross Output (GNR)	Gross Output (OLS)	Gross Output (GNR)	Gross Output (OLS)	Gross Output (GNR)
Labor	0.15	0.21	0.21	0.31	0.32	0.39	0.32	0.42	0.29	0.42	0.26	0.35
	(0.01)	(0.01)	(0.02)	(0.03)	(0.01)	(0.02)	(0.03)	(0.04)	(0.02)	(0.02)	(0.01)	(0.01)
Capital	0.04	0.09	0.06	0.14	0.01	0.06	0.03	0.02	0.03	0.09	0.06	0.13
	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.03)	(0.01)	(0.01)	(0.00)	(0.00)
Intermediates	0.82	0.71	0.76	0.56	0.68	0.53	0.65	0.53	0.73	0.54	0.72	0.56
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.00)	(0.00)
Sum	1.01	1.01	1.03	1.01	1.01	0.98	1.00	0.96	1.05	1.06	1.04	1.04
	(0.00)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)	(0.04)	(0.01)	(0.01)	(0.00)	(0.00)

Chile

Labor	0.17	0.27	0.26	0.42	0.29	0.44	0.20	0.38	0.32	0.48	0.20	0.35
	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)
Capital	0.05	0.10	0.06	0.10	0.03	0.06	0.02	0.06	0.07	0.13	0.09	0.15
	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)	(0.00)
Intermediates	0.83	0.69	0.75	0.57	0.74	0.58	0.81	0.62	0.71	0.53	0.77	0.59
	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)	(0.00)
Sum	1.05	1.06	1.06	1.09	1.06	1.08	1.04	1.06	1.10	1.13	1.06	1.08
	(0.00)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)	(0.00)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers in the first column are based on a gross output specification of a translog approximation to the production function and are estimated using OLS. The numbers in the second column are also based on a gross output specification, but estimated using our approach.