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Indexed Sovereign Debt: An Applied Framework*

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Abstract

In recent years, some countries have issued sovereign bonds indexed to real variables such as GDP. Moreover, there has been discussions about this issue during the European crisis. This paper analyzes the effects of introducing this type of contracts in a standard DSGE model with sovereign default risk. We solved the model numerically calibrating it to the Argentine economy and show that the introduction of GDP-indexed sovereign debt contracts reduces the probability of default and makes the government willing to hold non-contingent assets and issue real-indexed bonds at the same time. The magnitude of the welfare effect that this type of instruments could generate is equivalent to an increase of approximately half a percentage point per year in certainty equivalent aggregate consumption.

JEL codes: E6, F41, G15

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1 Introduction

In March 2005 Argentina finished the debt restructuring process that followed the sovereign default and financial crisis of 2001. In the restructured debt, a sizeable share of the payments were linked to the future evolution of Argentina's GDP. This captured the spirit of some of the proposals under discussion since the string of sovereign debt crises in Russia, Ukraine, Pakistan, Ecuador and Argentina renewed the debate on the institutional framework of sovereign credit markets and the design of sovereign debt contracts.[1]

A contingent contract that stipulates lower payments to creditors when a bad shock hits the economy, and higher payments when a good shock occurs, could improve risk sharing between debtor countries and international creditors and diminish the probability of debt crises. Indexing bond payments to real variables such as commodity prices or GDP (i.e. real-indexed bond contracts), as Argentina did, could achieve this and, as a result, generate welfare gains.[2]

The recent international financial crisis has led some policymakers to contemplate for the first time the possibility of issuing debt with such features.[3] In a fragile world economic environment, in which many developed countries are facing a default, it is of special interest to evaluate quantitatively the effects of introducing real-indexed sovereign bond contracts.

This paper analyzes how the introduction of real-indexed sovereign debt contracts affects sovereign governments' borrowing, asset holding and default decisions, and welfare. We do this by introducing this type of contract in a standard dynamic stochastic general equilibrium (DSGE) model with endogenous sovereign default risk, as the ones developed in Aguiar and Gopinath (2006) and Arellano (2008).

While we introduce these contingent sovereign debt contracts, we assume that the set of assets that the government can hold is identical to that of most standard sovereign debt models, i.e. non-contingent assets that yield the risk free rate. This generates an interesting result as, in our model, the presence of real-indexed debt contracts makes the government want to hold non-contingent

[1] See IMF (2002) for a summary of some of these proposals.

[2] See Borensztein and Mauro (2002) for an informal discussion of these effects.

[3] See for instance "Irish debt repayments should be linked to growth" by the governor of the Central Bank of Ireland Patrick Honohan , Financial Times, April 6 2011.

assets and issue contingent debt at the same time. The reason why this happens is that contingent debt and non-contingent assets span different state spaces.[4]

Our calibration of the model with GDP-indexed debt to the Argentine economy for the period 1983-2000 shows that with GDP-indexed debt the government borrows more and, simultaneously, default is less likely than with non-contingent debt. We also show that welfare gains from substituting standard defaultable non-contingent sovereign bonds with GDP-indexed bonds would be equivalent to an increase in average aggregate consumption of approximately 0.5% per year. This welfare gain is higher if the government is more impacient.

The basic structure of the model is the following. There is a small open economy where the government tries to maximize the welfare of an infinitely lived, risk averse, representative agent facing random income shocks. In order to do so, the government can borrow from international creditors using one period GDP-indexed bonds that promise payments to creditors increasing in the level of GDP.[5][6] The government can also save using non-contingent assets that yield the risk free interest rate. Consistently with the weak legal framework of sovereign borrowing, we assume that the government cannot credibly commit to repay its debts to foreign creditors. Every period it chooses whether to repay or default on its outstanding debts. Default entails direct costs or penalties and triggers limited time exclusion from credit markets.[7] Government borrowing will be endogenously constrained by the presence of limited commitment.

In this framework, the introduction of GDP-indexed debt contracts improves overall welfare of the borrowing country through two channels: first, it increases the amount of risk-sharing between a risk-averse sovereign borrower and risk-neutral lenders, and, second, it reduces the probability of default. Welfare improves even if we take into account that non-indexed debt contracts are made contingent through default in the presence of limited commitment. However, a default does so in a costly way.

[4] This does not happen in standard models in the sovereign debt literature such as Aguiar and Gopinath (2006) or Arellano (2008) because they implicitly assume that assets held by the government are seized upon default. In our model instead, the fact that debt is contingent and assets are not, allow the government to span a different state space by holding assets and debt compared to just holding debt.

[5] This assumption is consistent with existing indexed sovereign debt contracts. See the Appendix for details.

[6] For the sake of realism, we assume that the contract can only stipulate non-negative payments to creditors.

[7] These direct costs of default or penalties have been interpreted as trade sanctions by Bulow and Rogoff (1989), as reputation spill-overs by Cole and Kehoe (1997) and as informational costs by Sandleris (2008).

Some policymakers have been concerned that sovereign debt contracts indexed to real variables partially under the control of the government, such as reported GDP, may create or worsen moral hazard.[8] In effect, the government may have incentives to undertake less growth-oriented policies or underreport GDP growth rates if interest payments increase in GDP. However, the presence of moral hazard is not exclusive to indexed contracts. For example, if the costs of default are affected by the level of income, distorsions on the government policies might potentially also arise with non-indexed debt too. Moreover, the extent and magnitude of this kind of moral hazard is not clear in practice.[9]

Our paper bridges two branches of sovereign debt literature that have, surprisingly, remained separate. The first strand emphasizes the presence of limited commitment in a dynamic framework. Within this group our model is related to Eaton and Gersovitz (1981) and, in particular, to the adaptations of their model made by Aguiar and Gopinath (2006), Alfaro and Kanczuk (2007) and Arellano (2008). We extend this framework to study real indexation of sovereign debt contracts and their quantitative welfare effects. The second strand of literature, which is more policy oriented and best represented by Borensztein and Mauro (2004) and Durdu (2008), analyzes real-indexed sovereign debt contracts abstracting from a thorough modelling of the limited commitment structure.

The paper is organized as follows. Section 2 presents our baseline model. Section 3 calibrates the model to the Argentine economy and assesses its quantitative implications. Section 4 concludes.

[8] Krugman (1988) was the first one to point this out. In order to address this issue, some studies (see for instance Caballero (2002)) have proposed to index the contracts to variables beyond the control of the government, such as commodity prices or trading partners' growth rates.

[9] There are many examples of inflation indexed debt that are actively traded and, arguably, governments have a stronger incentive to under report inflation than GDP. While lower reported inflation and GDP growth rates would lead to lower interest payments on indexed contracts, lower inflation is perceived to be a good signal while the opposite happens with lower GDP growth rates. Thus, if moral hazard is an issue, it should already prevent the existence of inflation indexed debt. This is not what we observe in reality, so we will abstract from it in this paper.

2 The Model

2.1 Environment

Consider a small open economy in the spirit of Eaton and Gersovitz (1981) where a benevolent government tries to maximize the welfare of an infinitely lived representative agent. In each period $t = 0, 1, 2, 3, ...$ the agent receives a stochastic endowment, $y_t(s)$, contingent on the state of the economy, $s \in S$, a compact subset of $\mathbb{R}_+$, where $y_t(s+i) > y_t(s)\ \forall i > 0$. This endowment follows a Markov process with transition density $f(s', s)$. The preferences of the representative agent are given by the following sum of instantaneous utility functions:

$$U = \sum_{t=0}^{+\infty} \beta^t E_0 \left[u(c_t)\right] \tag{1}$$

where $\beta < 1$ represents the intertemporal discount factor, $u(.)$ is a strictly concave, continuous and differentiable utility function increasing in c_t, consumption in period t, and E_0 is the time 0 expected value of the utility from consumption in future periods.

The key element that distinguishes our model from standard sovereign debt models such as Aguiar and Gopinath (2006) or Arellano (2008) is that in our model the government can issue one-period debt indexed to the state of the economy. Formally, this debt, d_t, promises state contingent payments $x(s)$ the following period. For the sake of realism we assume that promised payments are non-decreasing in the state of the economy.[10] Formally, $x(s) \geq 0 \forall s$ and $x(s+i) \geq x(s)\ \forall s, i > 0$. Note that a non-contingent discount bond can be thought of as a particular case of our bond contract, one in which $x(s) = 1 \forall s$.

The government also has access to a risk free one-period international asset, a_t that has a price of 1 and pays R_t^w in every state. This asset can be thought of as international reserves. In every period t, after observing the state of the economy s, the government makes two decisions in order to maximize the welfare of the representative agent: whether to repay or default on its outstanding debt with foreign creditors, and, if it decided to repay, how much new debt to issue and assets to purchase.

[10] See Appendix for a detailed analysis of the characteristics of existing real-indexed debt contracts.

The promised state contingent payments *to* the rest of the world generated by the government portfolio of assets and liabilities can be written as:

$$b_t(s) = x(s)d_t \square R_t^w a_t \tag{2}$$

where $d_t \geq 0$ and $a_t \geq 0$. With a little abuse in definition, we will sometimes refer to $b_t(s)$ as the level of net debt of the economy. The state contingent budget constraint for the economy at time t in contingency s then becomes:

$$\begin{aligned} y_t(s) + h_t \left[q_t(x(s\prime)d_{t+1}, a_{t+1}, y_t(s))d_{t+1} \square a_{t+1} \right] + R_t^w a_t = \\ = c_t(s) + h_t(s)x(s)d_t + \lambda(h_t(s), y_t(s)) \end{aligned} \tag{3}$$

where $y_t(s)$ is national income and $q_t(x(s\prime)d_{t+1}, a_{t+1}, y_t(s))$ is the unit price of debt issued at time t that in equilibrium depends on the amount of debt issued, the state contingent payments promised and the assets held by the goverment, and on the current national income; $h_t(s)$ denotes the choice of the country between repaying and defaulting at time t in state s:

$$h_t(s) = \begin{cases} 1 \text{ if the country repays} \\ 0 \text{ if the country defaults} \end{cases}$$

$\lambda(h_t(s), y_t(s))$ represents the direct cost (or penalty) to the country for not meeting its current payment obligations, $x(s)d_t$. We remain agnostic regarding the exact nature of these costs of default but we assume that $\lambda(h_t(s), y_t(s))$ is a continuous, differentiable and monotonically non-decreasing function on the level of income, $y_t(s)$, with $0 < \frac{\partial \lambda(0,y)}{\partial y} < 1$.[11] As it is standard in this literature the costs that arise in the case of default are not appropriated by the creditors. There is no penalty when the country pays back its debt obligations in full:

$$\lambda\left(1, y(s)\right) = 0, \forall s$$

Consistently with the empirical evidence, we assume that countries are temporarily excluded from credit markets after default. The probability that the country re-enters international financial markets in any subsequent period is exogenously given and equal to θ. As it is standard in this

[11] This assumption is standard in the sovereign debt literature and goes back to Grossman and Van Huick (1988) argument of excusable and non-excusable defaults. An alternative interpretation for this assumption can be found in Sandleris (2008)

literature, while excluded from credit markets, the government cannot borrow, purchase or hold assets. In other words, the government is forced to "consume" the assets it already holds in the period in which it defaults, otherwise they will be seized.

International lenders are assumed to be risk neutral (or, alternatively, able to fully diversify country risk), so they only consider the expected rate of return paid by the country's securities. Additionally, international credit markets are competitive. Therefore, in equilibrium the expected return from the sovereign debt is equal to the world itnerest rate:

$$E_{t\square 1}\left[R_t\right] = \frac{\sum_s \pi_t(s) h_t(s) x(s)}{q_{t\square 1}} = R_t^w \tag{4}$$

where $\pi_t(s)$ is the probability that contingency s realizes.

2.2 Value functions and recursive equilibrium

The benevolent government optimizes as follows:

$$\begin{gathered} \max_{c_t(s),h_t(s)=\{0,1\},d_{t+1}(s),a_{t+1}(s)} \sum_{t=0}^{\infty} \beta^t E_s\left[u(c_t(s))\right] \\ s.t. \\ c_t(s) = y_t(s) + h_t(s)\left[q_t(x(s\prime)d_{t+1}(s), a_{t+1}(s), y_t(s))d_{t+1}(s) \square x(s)d_t \square a_{t+1}(s)\right] + \\ + R_t^w a_t \square \lambda(h_t(s), y_t(s)) \end{gathered} \tag{5}$$

Our assumption on credit markets exclusion implies $a_{t+1} = d_{t+1} = 0$ if the country defaults.

Our setup allows for a recursive formulation. The state variables of the recursive problem are three: the income level, $y_t(s)$, the amount of debt payments, $x(s)d_t$, and asset holdings, a_t. We can write the value function for the government problem as:

$$V(y(s), x(s)d, a) = \max_{h=\{0,1\}} \left\{ V^{h=1}(y(s), x(s)d, a), V^{h=0}(y(s), a) \right\} \tag{6}$$

where $V^{h=1}(y(s), x(s)d, a)$ and $V^{h=0}(y(s), a)$ are the value functions of repaying and defaulting respectively, which are given by the following expressions:

$$V^{h=1}(y(s), x(s)d, a) = \max_{a', d'} \left\{ u(y + qd' + R^w a \square x(s)d \square a') + \beta E\left[V(y(s\prime), x(s\prime)d\prime, a\prime)\right] \right\} \tag{7}$$

$$V^{h=0}(y(s),a) = \left\{ u(y + R^w a \square \lambda(0, y(s))) + \beta E \left[(1 \square \theta) V^{h=0}(y(s\prime), 0) + \theta V(y(s\prime), 0, 0) \right] \right\} \quad (8)$$

where we adopt the convention of denoting with "$\prime$" a variable referred to "$t+1$". Notice that, while the government is excluded from the credit market, it only consumes its income and cannot make any other choices.

The government default policy can be characterized by repayment and default sets. For a given level of assets, a, and promised payments, $x(s)d$, we can define the repayment and default sets as follows:

$$I(y(s)) = \{y(s) : V^{h=1}(y(s), x(s)d, a) \geq V^{h=0}(y(s), a)\}$$
$$O(y(s)) = \{y(s) : V^{h=0}(y(s), a) > V^{h=1}(y(s), x(s)d, a)\}$$

The recursive equilibrium of this economy can be defined as follows:

Definition 1 *The recursive equilibrium of the economy is defined as: (i) a set of policy functions for the government $\{c(s), d'(s), a'(s), I(y(s)), O(y(s))\}$; and (ii) a sovereign bond price function $q_t(x(s\prime)d\prime, a\prime, y(s))$ such that:*

1. *taking the bond price function $q_t(x(s\prime)d\prime, a\prime, y_t(s))$ as given, the government policy functions $\{c(s), d'(s), a'(s), I(y(s)), O(y(s))\}$ maximize $V(y(s), x(s)d, a)$;*
2. *$q_t(x(s\prime)d\prime, a\prime, y_t(s))$ is consistent with creditors' zero expected profit condition (4) and reflects the government default probabilities.*

2.3 Sovereign Defaults with Indexed Debt

In this subsection we analyze how the debt level and the level of income affect the government default decision.

Lemma 2 *For a given level of assets, a, let $d_1 \leq d_2$, if default is optimal for d_1 in some states s, it will also be optimal for d_2 in the same states s.*

Proof. It follows from Proposition 1 of Arellano (2008) ■

The intuition for this result is that the cost of repaying increases in the amount of debt, while the cost of defaulting remains unchanged with it. So, for given state contingent promised payments $x(s)$ as the level of debt goes up, there are more incentives to default.

This implies that, as in Arellano (2008), the probability of default is increasing in the level of debt, and as a result, the bond price is decreasing in it. This means that there will be a threshold level of debt payments above which the default set will include the entire support of the income distribution. Clearly, creditors will never choose to lend such amount, as they would receive no payments in all possible contingencies. The level of government borrowing is therefore restricted in equilibrium by its lack of commitment to repay.

Once we introduce contingent payments, it is not the case that if the country chooses to default in state s with income $y(s)$, it will also default in any state $s \square i$ with income $y(s \square i)$. The reason is that an indexed contract may reduce the promised payment a lot when the economy is in a bad state (in the extreme, it may call for no payments at all in such a state), and, as a result for a given level of debt, the government may choose not to default for the lowest income levels, but yes for higher ones.

3 Calibration and Quantitative Analysis

In this section we solve the model numerically calibrating it to the Argentine economy. In doing so we analyze how the introduction of indexed sovereign debt contracts affects the government borrowing, assets holdings, and default decision. We also analyze the impact of introducing these contracts on welfare comparing the welfare of the economy under two types of debt arrangements: one in which the government can only issue standard zero coupon bonds so that promised payments are constant across states, and another in which it can only issue contingent debt where promised payments are monotonically increasing in the state of the economy. Under both debt arrangements the government can also purchase an international asset that makes non contingent interest payments.

3.1 Calibration

Argentina is the benchmark economy in our quantitative exercise. We choose Argentina because the country experienced one of largest of sovereign defaults in recent history and it also issued GDP-indexed sovereign debt. The GDP statistics for Argentina correspond to the seasonally adjusted quarterly real GDP series for 1983.1-2000.2 obtained from the Ministry of Economy and Production (MECON) of Argentina. The series are filtered with the Hodrick-Prescott procedure. Output, debt and private consumption are expressed as a percentage of GDP. We choose the parameters based on existing empirical work about emerging markets. Otherwise parameters are set to mimic empirical regularities of the Argentine economy. The per period utility function of households is assumed to have a CRRA specification:

$$U(c) = \frac{(c)^{1\square\sigma}}{1\ \square\ \sigma} \tag{9}$$

with a standard value for the risk aversion coefficient, i.e. $\sigma = 2$. The quarterly risk-free interest rate is set at 1%, which corresponds to the US real annual interest rate. Consistent with the empirical findings of Gelos, et al (2003) on credit market exclusions for defaulting countries in the 90s, the reentry probability, θ, is set to 0.2, which corresponds to about one year of exclusion. The discount factor, β, is set at 0.95 to match the 4.4% average external debt service to GDP ratio of Argentina for 1983-2000 based on World Bank data. A high degree of impatience has been linked to political factors in emerging economies (see, for instance, Cuadra and Sapriza (2006) and Hatchondo, Martinez and Sapriza (2006)).

The parameters characterizing the income process in the model are set to approximately match the observed volatility of 4.08% of the cyclical component of GDP in Argentina for the period under study. The output follows an AR(1) process with an autocorrelation coefficient of $\rho = 0.9$ and $\sigma_z = 3.4\%$. The mean of log output equals 0 so that average detrended output in levels is standardized to one. We solve the model numerically using the discrete state-space method. Each period represents a quarter. The process of output is approximated by a discrete first order Markov chain with 9 values using Hussey and Tauchen's (1991) procedure.

As a benchmark, we assume that when the country defaults, it experiences a one period output

loss, λ, of 8% of the output in a quarter.[12] Arteta and Hale (2006), Sandleris (2008) and Dooley (2000) among others, provide a rationale for the loss of output when countries face debt crises. The complete model parameterization is shown in Table 1.

The indexation rule corresponds to an increasing function of output with a range that goes from 15% of the bond face value (set equal to 1 without loss of generality) for the lowest income level to 125% for the highest.

The welfare comparison between the indexed and non-indexed bond economies are made in terms of certainty equivalent consumption following the methodology of Lucas (2003). That is, we compare the certainty equivalent consumption computed under each of the two debt arrangements.

3.2 Simulation Results

In this section we describe some key quantitative properties of the model and quantifies the welfare gains that may arise from introducing indexed debt contracts. We also perform a sensitivity analysis with respect to key parameter values like the harshness of the output penalty and time-discounting.

In order to evaluate the importance of introducing indexed debt contracts in the model, we compare two economies. In the first one, only standard discount bonds are available; in the second one, only indexed bonds are available. Figures 1 and 2 show the default regions (i.e., the combinations of income shocks and foreign debt levels for which default is the optimal decision) for the economy with standard discount bonds (non-indexed debt) and indexed debt contracts respectively.

As we showed in the previous section, given an income realization and a level of assets, if default is optimal for a certain level of debt, it will also be optimal for all higher levels. Analogously, if repayment is optimal for a given debt level, it will be optimal for all lower levels. Thus, for each realization of the income process, default incentives are stronger for highly indebted governments. This result arises because the value of paying back and remaining in good credit standing is strictly decreasing in foreign debt, while the value of defaulting and going to autarky does not depend on the amount of foreign debt but only on income.

[12] This is approximately equivalent to an average output loss of 2% during the year in which the country is on average in autarky. This is consistent with the findings of Chuhan and Sturzenegger (2003) and the mean deviation from trend of Argentina's output of 7.3 percent while in a state of default found by Arellano (2008).

Comparing Figure 1 and Figure 2 we can also observe that the default regions under non-indexed debt and indexed-debt take the expected shapes. First, default is more likely with non indexed debt, i.e. a lower amount of debt can be supported for a given level of income. Second, the default choice tends to be less income dependent with an indexed debt contract that promises payments increasing in the state of the economy.[13]

As expected, for a given income realization the discount bond price schedule (not displayed) is a decreasing function of foreign debt. For small levels of foreign debt, the government always pays back its debt, so it borrows from international markets at the world risk free interest rate. In this debt range, the bond price is simply the inverse of the gross risk free rate. However, as the foreign debt increases, at a certain level the bond price starts to decrease because the incentives to default are stronger for highly indebted governments. At a sufficiently large debt level the government always defaults regardless of the output realization, so the probability of default is one and consequently the bond price is zero. The bond price schedule also illustrates that for all levels of debt, the bond price is lower (the interest rate is higher) when the economy is hit by an adverse output realization. Since productivity shocks are persistent, lending resources to the government in times of low output involves a higher default risk, and, as a result, a higher interest rate.

Figures 3 and 4 depict the corresponding portfolio choices - the amount of assets purchased and debt issued by the sovereign country - when simulating the benchmark economy defined in Table 1 for 5000 periods under both debt arrangements. Figures 5 and 6 display the optimal portfolio choices and present some sensibility results for this analysis for the case where β is increased to 99%, a value that is probably an upper bound in this litertature.

There are two main messages from these figures. First, with indexed debt contracts, the government issues optimally much more debt on average than with non-indexed debt (i.e. discount bonds). The intuition for this result is that with indexed debt, the government finds it esier to avoid default for any given level of debt and as a result, it is willing to engage in more borrowing.

The second important message is that the presence of indexed debt induces a country to optimally issue debt *and* purchase international assets at the same time much more frequently than

[13] An indexed debt contract could potentially be designed to yield a perfect rectangle for the default set.

when the debt contracts are standard dicount bonds.[14] The reason for this result is quite interesting and not immediately obvious: assets matter because, in conjuction with the indexed bond, they provide the optimal stream of payments *to* and *from* the country that stabilizes consumption as much as possible across contingencies. In other words, holding assets and issuing indexed debt at the same time allows the government to better insure itself (i.e. span a larger state space). On the contrary, when the country can only issue non-indexed debt, assets matter only in the contingencies when it defaults. As default events are relatively rare in our simulations, holding assets is much less attractive.

Table 2 shows the welfare gains that arise when a government can borrow using only indexed debt relative to a situation where only non-indexed debt is available. The welfare gains range from 0.1% to 0.5% in terms of certainty equivalent consumption and are the result of better risk sharing and less frequent defaults. This means that our simulated economy would be able to achieve a constant level of consumption nearly 0.5% higher (in perpetuity) when the debt contracts are indexed. This estimate was derived as follows. First, we compared the levels of utility of a country subject to the same income shock under the two alternative debt contracts (standard and indexed). Under each debt arrangement there were different distributions of debt and asset levels, default regions and consumption. We used each distribution to compute a level of utility corresponding to each debt contract. Finally, each level of utility can be used to derive, inverting the utility function, the level of consumption that, in perpetuity, would guarantee it. Following Lucas (2003), we define this consumption level as *certainty equivalent consumption*, the basic ingredient of the welfare comparison we propose.

Finally, we find that the welfare gains of having indexed debt contracts increase when the default penalty is harsher. The reason is that with indexed debt, defaults occur less frequently. Perhaps more surprisingly, we find that more impatient governments benefit more by the introduction of debt indexation. The intuition for this result is that with indexed debt, the frequency of default falls substantially. This generates a large increase in the amount of debt that can be supported in equilibrium. As a result, a relatively impatient government is able to front load more consumption.

[14] The fact that the government does not want to hold assets when only the non-indexed debt contract is available is consistent with the findings of Alfaro and Kanczuk (2009).

4 Conclusions

Proposals on real indexation of sovereign debt contracts have been around for some time. During recent years, a number of countries have issued these kind of instruments, and the recent global financial crisis has prompted some european officials to consider the possiblity of having their countries issue real GDP indexed sovereign debt. However, there was no research analyzing the effects on borrowing, assets holdings, and welfare of introducing these contracts in a dynamic equilibrium framework. This paper fills such void.

We develop a dynamic stochastic small open economy model with real-indexed debt and show that with this kind of instruments, governments would optimally choose larger levels of debt, and that holding assets and issuing indexed debt at the same time could be optimal. We also show that indexed debt makes defaults less frequent and improve welfare. Furthermore, our study provides the first model-based quantitative assessment of their welfare effects. The improvement in welfare is equivalent to an increase in the order of 0.5% in aggregate average consumption. The magnitude of the welfare gains is positively correlated with the direct costs of default and negatively related to the time discount factor.

Finally, we leave for future research two issues that are beyond the scope of the present paper. The first one is related to the optimal design of real-indexed contracts. The second one is the issue of moral hazard that has received some attention in the literature on indexed debt. Tackling them could help to improve the quantitative assessment of the impact of indexed debt and may bring new insights on the design of real indexed debt contracts.

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6 Appendix

6.1 Algorithm

1. Assume an initial function for the price of the bond $q_0(a', xb', s)$. Use the inverse of the risk free rate.

2. Use q_0 and the initial values of the value functions (eg. start with 0 matrices) to iterate on the Bellman equations and solve for the value functions and the policy functions for assets and default.

3. Given the default function and the default sets, update the price of the bond using the following equation:

$$q_{t\square 1}(s) = \frac{\sum_s|_{h_t(s)=1} \pi_t(s) \cdot x(s)}{R^w} \tag{10}$$

4. Use the updated price of the bond to repeat steps 2 and 3 until the following condition is satisfied:

$$\max\left\{q_i \overset{\square}{}(a, xb', s) \square q_{i\square 1} \overset{\square}{}(a, xb', s)\right\} < \epsilon \tag{11}$$

where i represents the iteration and ϵ is an arbitrary small number.

6.2 Tables and Figures

Table 1. Benchmark Parameter Values

Discount Factor	β	0.95
Risk Aversion	σ	2
Re-entry Probability	θ	0.20
Output Loss	λ	8%
Risk Free Interest Rate	r_f	0.01
Autocorrelation coefficient of innovations	ρ	0.90
Standard deviation of innovations	σ_z	0.034

Table 2. Welfare Comparizon: GDP-Indexed Vs Standard Debt			
(% Gain in Terms of Certainty Equivalent Consumption)			
		β	
$\lambda(y)$	0.8	0.93	0.95
a la Arellano (2004)			
8%	0.51%	0.17%	0.15%
4%	0.39%	0.12%	0.11%
Average credit markets exclusion: 4 quarters			
Indexed Debt $\overrightarrow{x}$: Vector of State Contingent Interest Payments			
$x[y(s)] = [0.15, 0.2, 0.25, 0.3, 0.6, 1, 1.1, 1.15, 1.25]$			

6.3 A Brief Survey of Real-Indexed Sovereign Debt contracts[15]

Proposals to index sovereign debt payments to real variables have been around for nearly 25 years when the debt crisis of the 1980s triggered interest in the issue. Around that time, Bailey (1983) suggested that debt should be converted into claims proportional to exports, and Lessard and Williamson (1985) made the case for real indexation of debt claims. A few years later, Shiller (1993) discussed the importance of creating "macro markets" for perpetuities linked to GDP.

The recent string of sovereign debt crises in Russia, Ecuador, Pakistan, Ukraine and Argentina generated a second wave of interest in contingent sovereign debt contracts for emerging countries. Haldane (1999), Daniel (2001) and Caballero (2003) suggested that countries would benefit from issuing debt indexed to some relevant commodity price. Drèze (2000) suggested the use of GDP-indexed bonds as part of a strategy to restructure the debt of the poorest countries and Borensztein and Mauro (2002, 2004) tried to revive the case for GDP-indexed bonds for emerging countries.[16]

The basic idea behind all these proposals is to use indexed sovereign debt as a way to improve risk sharing between debtor countries and international creditors and, in doing so, to reduce the

[15] This section is provided for completeness and to the benefit of editor and referees. Its aim is to describe the relatively large set of existing indexed debt instruments. It can be dropped at the editor/referees discretion.

[16] See Borensztein and Mauro (2002) and Borensztein et al. (2004) for a detailed analysis of indexing proposals for sovereign debt.

probability of occurrence of debt crises. One important difference across proposals is whether they suggest indexing the debt instruments to variables partially under the control of the government or beyond it. While indexation to broader measures such as GDP or exports that are partially under the control of the government would likely provide greater insurance benefits, potential investors might be concerned about the authorities' incentives to tamper with data or undertake less growth-oriented policies. These concerns regarding the potential risks of moral hazard were first discussed in this context by Krugman (1988) and Foot, Scharfstein and Stein (1989).

It is not clear how relevant are these moral hazard issues in reality. However, if they were, the option of indexing debt contracts to commodity prices outside the control of the government would only be useful to a small group of emerging countries as GDP growth is poorly correlated with these variables for most of them.

Although more than 20 developing and developed countries have issued inflation indexed debt, the experience with bonds indexed to real variables is much more limited. Table 1 summarizes the experiences with each of them.

Table 3: Countries Issuing Real Indexed Bonds

Type	Country
GDP-Indexed Debt	Costa Rica (1990), Bosnia and Herzegovina (1990s), Bulgaria (1994), Argentina (2
Commity Indexed Debt	US (1864), France (1970s), Mexico (1990s), Nigeria (1990s) Venezuela (1990s)

Source: Borensztein et al. (2004)

6.3.1 Argentina GDP-warrant

The most recent experience with sovereign bonds indexed to real variables is Argentina's GDP-warrant. In March 2005, Argentina finished its debt restructuring process that followed the default and financial crisis of 2001. Each new bond issued in the restructuring included a unit of GDP-linked warrants. These warrants were tied to the bonds for the first 180 days, and became detachable thereafter.

Given the magnitude of the restructuring, Argentina's GDP-warrants are the first sovereign

debt instruments indexed to real variables for which there is a sizable market. The GDP-linked securities have a notional amount equal to the corresponding defaulted debt tendered and accepted in the 2005 restructuring, converted to the corresponding currency using the exchange rate as of December 31, 2003 (roughly US$62 bn, of which AR$88 bn, US$14.5 bn external and US$2.9bn internal, EUR11.9 bn).

Payments on the GDP-linked securities take place only if the following three conditions are met:[17]

(i) Actual real GDP exceeds the base case GDP for each reference year

(ii) Annual growth in actual real GDP is higher than the growth rate in the

base case GDP for the reference year (base case GDP real annual growth rate is 3.5% per year initially and gradually converging to 3%)

(iii) Total payments made on the security do not exceed the payment cap

of 48% of the notional amount during the life of the security.[18]

Whenever these three conditions are met the formula used to calculate the payments for each notional unit of the warrants is the following:

$$Payment = 0.05 \cdot (\text{Excess } GDP) \cdot (\text{Unit of Currency Coefficient})$$

where Excess GDP is the amount by which actual real GDP converted into nominal GDP using the GDP deflator exceeds the base case nominal GDP, and the Unit of currency coefficient is defined as: USD: $1/81.8 = 0.012225$, Euro: $1/81.8 \cdot (1/0.7945) = 0.015387$, ARS: $1/81.8 \cdot (1/2.91750) = 0.004190$.

As GDP data is usually published with a lag of a couple of months and it is usually revised in the following months, payments are calculated on the November following the relevant reference year, and made effective a month later. This creates a lag of a year between the economic performance

[17] In all cases calculations are based on the data published by Argentina's Bureau of Statistitcs (INDEC).

[18] The GDP-warrants expire when the 48 cents per dollar cap is reached, and no later than December 15th, 2035.

that might trigger a payment and the payment itself. Thus, potentially, troublesome situations may arise for the government. For example, assume that after a year of very high growth that meets all the conditions for the payment, an adverse shock drives the economy into a recession. The lag in the payments of the GDP warrant implies that the government will have to make a large payment precisely during the recession.

Trading of GDP-warrants began in a "when and if" market before they were detached. In May 2005 the "consensus" value of the GDP-warrant with dollar coupon among investors was 2 cents per dollar. In July 21st, the first available date with data from the "when and if" market, the bid price for the GDP warrants was 3 cents per dollar coupon (50% increase in two months). Furthermore, by the end of 2005 the Argentina's outstanding growth rates and a better understanding of the instrument led the markets to reevaluate their assessment of the value of the GDP-warrant. The price almost tripled when compared with the consensus value upon issuance. By the end of 2006 its price reached 13 cents per dollar coupon, six times higher than the "consensus" value at the time of the exchange and four times higher than the first available trading price. This remarkable increase in price has caught the attention of investors, authorities and the public in Argentina.

The first payment of the GDP-warrant took place in December 2006 and amounted to US$387 mn. In fact, given current consensus forecast for GDP growth in Argentina in the next couple of years, payments on the indexed component are expected to triple in the next two years. In 2008 payments of the indexed component are expected to be roughly equivalent to total coupon payments (plus capitalization) on the three new bonds issued in the restructuring considered together.

Summarizing, despite little interest shown initially by investors, the market for Argentina's GDP-warrant took off fueled by the excellent performance of Argentina's economy in recent years. This is good news for GDP-indexed bonds as it would be the first successful case of such an instrument. However, it is inevitable to wonder whether it was a good idea to include them in the exchange from the point of view of Argentina, given that they did not seem to have any significant impact in the level of acceptance of the proposal.

6.3.2 Brady bonds with Value Recovery Rights (VRRs)

Some years before Argentina's experience with GDP-warrants, a handful of emerging market economies issued bonds with elements of real indexation in the past. Various Brady bonds issued by Bosnia and Herzegovina, Bulgaria, Costa Rica, Mexico, Nigeria, and Venezuela in exchange for defaulted loans in the early 1990s included Value Recovery Rights (VRRs). The VRRs were designed to provide the banks with a partial recovery of value lost, as a result of the debt and debt-service reduction contemplated in the Brady exchange, in the event of a significant increase in the debtor country's capacity to service its external debt. Mexico's VRRs, for example, provided for the possibility of quarterly payments, beginning in 1996, based upon certain increases in the price of oil.

Brady bonds issued by Bulgaria, Bosnia & Herzegovina and Costa Rica contained elements of indexation to GDP. In the case of Bulgaria for example, its Discount Brady bonds had a component named Additional Interest Payments (AIP) that was indexed to GDP. The AIP was triggered when two conditions were met: (1) Bulgaria's GDP surpasses 125% of its 1993 level, and (2) there is a year-over-year increase in GDP. For these years (not including the year in which the threshold is reached) the semi-annual interest supplement was defined as half of that year's GDP growth. The outlays themselves were scheduled to occur "as soon as practically possible" and were to coincide with regular interest payment dates. The AIP were not warrants, detachable or otherwise, though they were intrinsically equivalent.

Bulgaria's GDP-linked bond was generally viewed as a failure as a result of two factors. First, the bonds were callable at par. This meant that the government could decide to repurchase the bonds rather than pay out when faced with onerous GDP-linked payments, and as a result investors would miss out on the lucrative upside. In fact this is exactly what happened. A second problem with Bulgaria's bonds was that the conditions were fairly vague. In effect, "GDP" itself was not well defined, so it was open to interpretation the exact measure of GDP to be used. The government made use of this ambiguity for a while choosing definitions of GDP that prevented the AIP from being triggered.

Bosnia and Herzegovina's GDP-linked Brady bonds included additional interest payments

whenever GDP growth rates exceeded for two years a predetermined growth rate and GDP per-capita rise above $2,800 (1997 USD, adjusted by German CPI). These bonds have also suffered with problems in the definition of GDP and their trading activity has been very limited according to Bear Stearns.

In general, the experience with VRRs has not been positive. Indexation formulas were complex and often ambiguous. There were restrictions on their tradeability and many times were not detachable, and some of the bonds were callable.

6.3.3 Commodity-Linked Bonds

The main advantages of bonds indexed to commodity prices are that the data is available without a time lag and is not subject to manipulation by governments. Compared with GDP-linked bonds, however, their main disadvantage is that for most countries the correlation between economic performance and commodity prices is relatively low.

Bonds whose repayments are indexed to commodity prices have been used, although rarely, since the 1700s. In 1782, the State of Virginia issued bonds linked to the price of land and slaves. In 1863 the Confederate States of America issued "cotton bonds", whose payments increased with the price of cotton. "Gold clauses," effectively indexing payments to the price of gold, were widespread in the United States in

the 19th century through 1933. France also experimented with gold-price-indexed bonds, the "Giscard," in 1973, but the losses caused by the depreciation of the French Franc caused the government to cease offering this instrument.

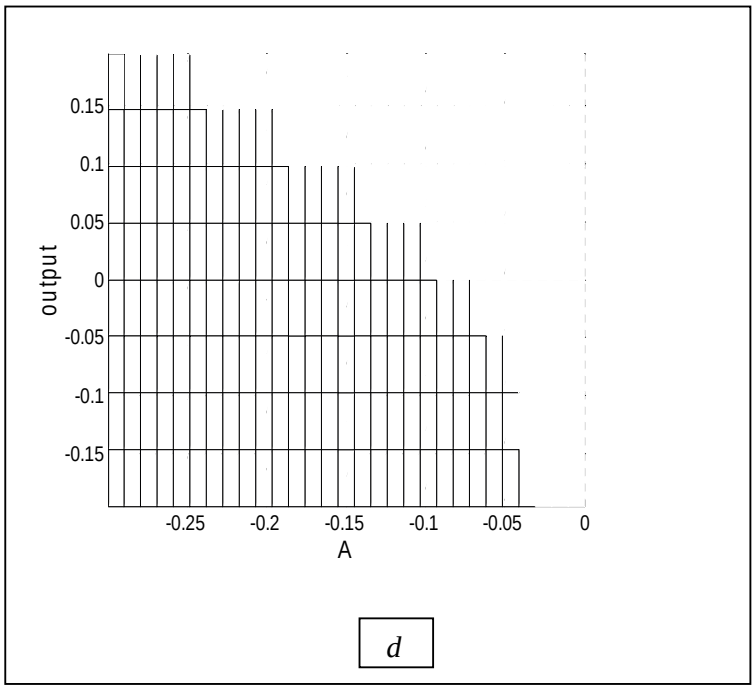
Oil-backed bonds appeared in the financial markets during the late 1970s. Mexico is considered the first country to offer oil-linked bonds in April 1977. The "Petrobonos" were issued domestically on behalf of the government by NAFINSA, a development bank owned by the Mexican government. They had a relatively active domestic secondary market developed in which most investors were Mexican. The bond promised to pay an annual rate of 12.65823% and had a three years maturity. Upon maturity, the Petrobonos were redeemed at a value equal to the maximum of the face value or the market value of the referenced units of oil plus all coupons received during the life of the

bond.

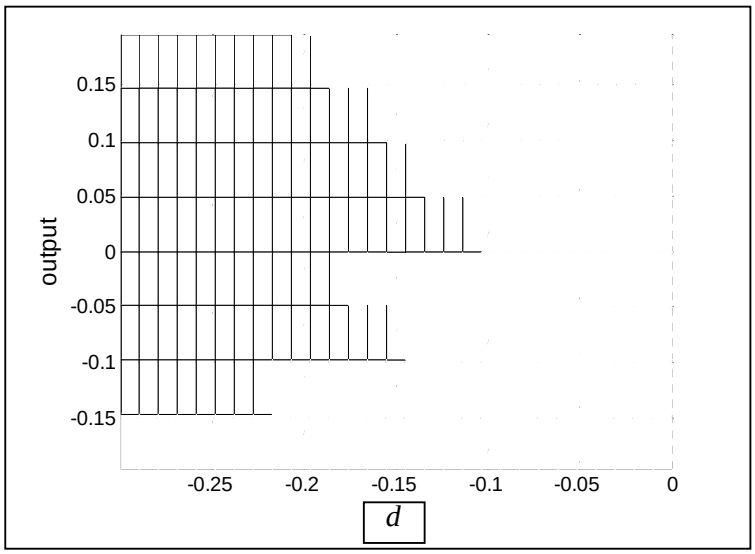
Other countries and private companies have also experimented with commodity-linked bonds. For instance, Venezuela issued oil linked-bonds as part of its Brady agreement. India issued oil linked bonds to oil companies in April 1998 in payment for debts it had incurred by receiving oil products below market cost. Malaysia accepted a loan from Citibank indexed to palm oil. More recently, loans combined with protection (through swaps) from commodity price fluctuations have also been made available by the World Bank to member countries, beginning in September 1999, though interest has thus far been limited.

Default Region

Benchmark Economy (β = 95%, θ = 20%, Default Penalty a la Arellano (2004) = 8%))
Indexed Debt: Vector of State Contingent Payments
x [y(s)] = [0.15 0.2 0.25 0.3 0.6 1 1.1 1.15 1.25]



Non indexed Debt - Figure 1

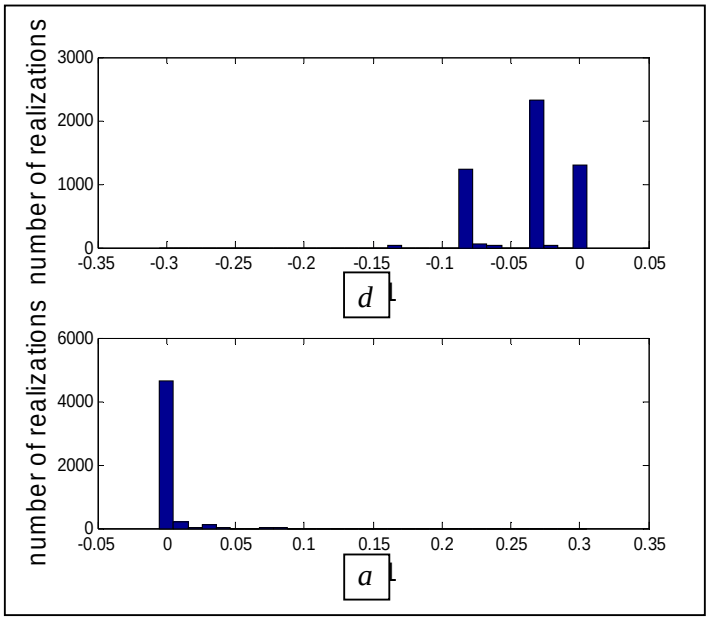


Indexed Debt - Figure 2

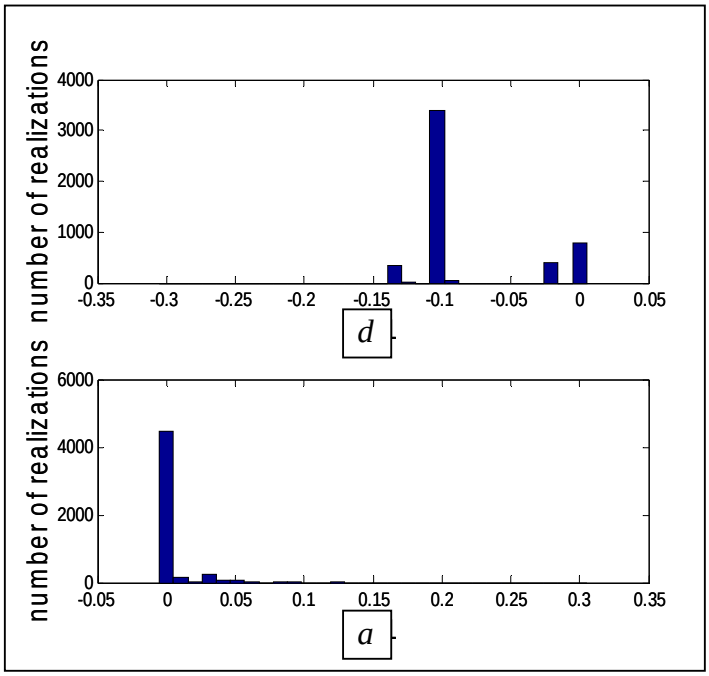
Country Optimal Portfolios

Number of Periods (over 5000 simulated periods)
in which the Country Undertakes the Displayed Asset/Debt Position

- Benchmark Economy (β = 95%, θ = 20%, Default Penalty *a la* Arellano (2004) = 8%))

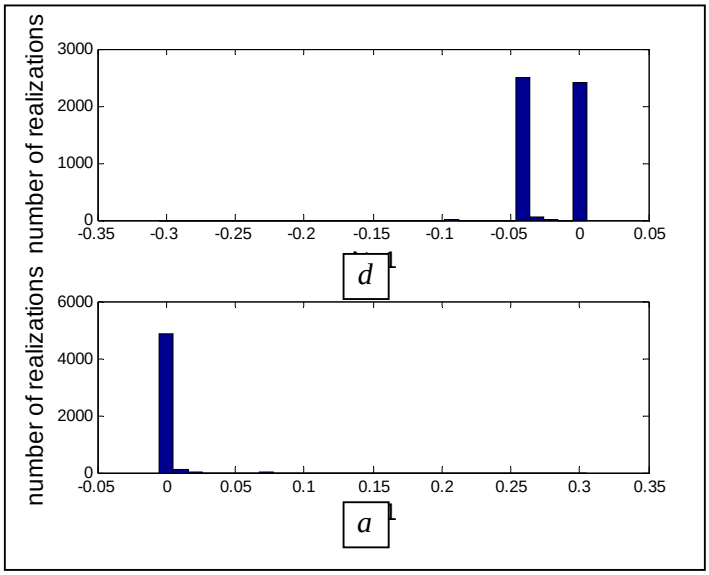


Non Indexed Debt - Figure 3

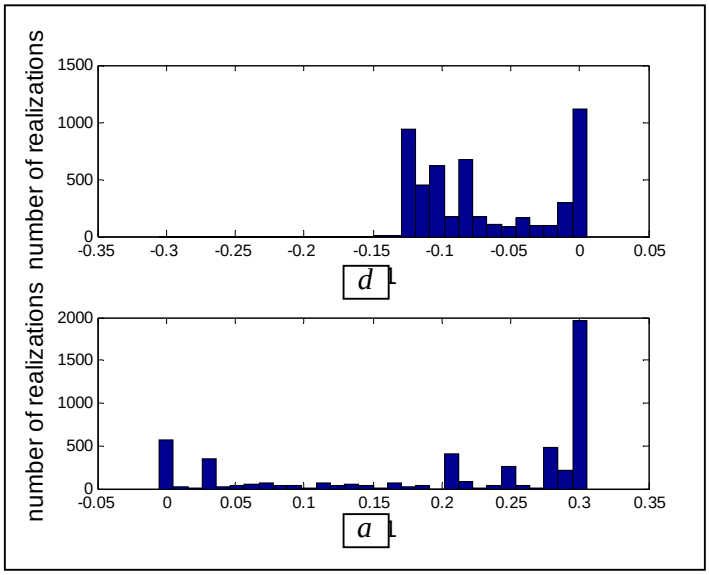


Indexed Debt - Figure 4

- β = 99%, θ = 20%, Default Penalty *a la* Arellano (2004) = 8%)



Non Indexed Debt - Figure 5



Indexed Debt - Figure 6